EFFECTS OF TEACHING THROUGH PROBLEM–SOLVING APPROACH ON STUDENTS' MATHEMATICS ACHIEVEMENT IN SECONDARY SCHOOLS OF MURANG'A COUNTY, KENYA

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DECLARATION AND APPROVAL

Declaration by the Student

This thesis is my original work and has not been presented for a degree in any other University for any other award.

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DEDICATION

To my wife Esther, and children Josephus, Brian, Mary, Jacqueline, Ann rose and Juliet

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ABSTRACT

Students' low performance in Mathematics at Kenya Certificate of Secondary Education (KCSE) in Murang'a County has been of concern to both primary and secondary stakeholders. There is need to look at this problem as students' performance in Mathematics determines their participation in science-oriented programmes at tertiary level. Many interventions been done to address this concern including the provision of Mathematics resource books, in-servicing of teachers of Mathematics through SMASSE programme but low students' achievement persists. There is a need then to explore other interventions. The current study investigated the effects of teaching through problem – solving on students' achievements in Mathematics in secondary schools of Murang'a County. The study objectives included:- (i) To determine extent, which teacher of Mathematics use problem–solving strategies in their teaching (ii) To compare students' performance in Mathematics for those taught using problem–solving strategies with those taught using conventional methods (iii) To assess students' change in attitude towards Mathematics where problem–solving strategies used instead of conventional methods in classrooms, and (iv) To develop a prototype lesson plan for problem–solving in teaching Mathematics The study employed a quasi – experimental design using Solomon Four Group design. The target population was 104562 students and 1365 Mathematics teachers in 340 public secondary schools in Murang'a County. The accessible population was 28475 Form Three students. The schools were stratified into four categories based on KCSE performances in Mathematics during a period of four years 2015-2017, giving an overall of 16 schools: eight schools for experimental group and eight schools for control group. The total sample size was 560 respondents comprising of 544 students' and 16 teachers. Students' Mathematics Attitude Questionnaire used to collect data on students' attitude towards Mathematics in both control and experimental groups. Four schools in experimental and four schools in control participated in pre-test and all sixteen schools received post-test Mathematics achievements tests after intervention. The collected data coded and analysed using Statistical Package for Social Sciences (SPSS) version 25. The statistical tests used to analyse data were t–tests, Analysis of Variance (ANOVA) and Cohen's d to establish the effect of problem–solving method of teaching Mathematics on students' achievement. Those who received pre-test had (M=39.36, n=128) experimental (E1) and (M=36.57, n=147) control (C1), show that students' performance pre-test was insignificant ($t\ (275) = 0.924$, and small Cohen's $d = 0.17$ ($P > 0.05$)). The post-test Mathematics Achievement revealed that experimental groups (E1+E2) performed better (M=47.57, n=254) against control groups (C1+C2) (M=36.56, n=290). The Cohen's d value of 0.71 showed a large effect size ($F\ (540) = 2.537$, $p = 0.0025$, $\alpha < 0.05$)). Problem–solving skills helped to change the students' attitude towards Mathematics (Cohen's d of 0.88, which is large. The study revealed that there was overall improvement on Mathematics achievement among the students taught using the problem-solving strategy. The study recommends that teachers should embrace problem–solving strategy rather than conventional methods in teaching Mathematics for better achievement in Mathematics in secondary schools of Murang'a County.

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LIST OF ABBREVIATIONS AND ACRONYMS

ANOVA	**:**	Analysis of variance
Df	**:**	Degrees of freedom
GM	**:**	Grand mean
KCSE	**:**	Kenya Certificate of Secondary Education
KNEC	**:**	Kenya National Examinations Council
KUCCPS	**:**	Kenya University and Colleges Central Placement Service
MoE	**:**	Ministry of Education
NACOSTI	**:**	National Commission for Science and Technology Innovation
NCTM	**:**	National Council of Teachers of Mathematics
SDG	**:**	Sustainable Development Goals
STDEV	**:**	Standard Deviation
SMASSE	**:**	Strengthening Mathematics and Sciences in Secondary Schools
TIMSS	**:**	Trends in International Mathematics and Science Study
VAR	**:**	Variance

CHAPTER ONE

INTRODUCTION

This chapter covers the following: background to the study, statement of the problem, the purpose of the study, objectives of the study, research hypotheses, justification of the study, the significance of the study, scope of the study, limitations and delimitations of the study, assumptions of the study and operational definition of key terms.

1.1 Background to the Study

Mathematics instruction and learning involves the learner, teacher and society. In order to achieve the intended aims and goals, the three must be involved (Lesh & Zawajewki, 2007). This study involved learners and teachers through problem - solving to obtain meaningful solutions to mathematical problems. The learner was involved directly in solving real-life problems. These done using problem-solving strategies in specified learning conditions in secondary schools. These conditions provide different learning situations under controlled classrooms.

According to Plato’s division of the liberal arts, education divided into a trio, which includes Mathematics, Literature, and Gymnasium. This historical perspective has shown that mathematical knowledge occupies the highest status in form of cognitive knowledge. These early fathers of Mathematics considered it the most essential subject because it teaches students how to think, how to be creative and resourceful by providing them with tools to use (Microsoft Encarta, 2009). Therefore, these early philosophers and mathematicians considered that the Mathematics discipline takes an important position in developing individual logical reasoning as well as playing a significant part in enhancing a country’s socio-economic development. This means that community drives in our daily activities involve the application of Mathematics. These social functions comprise merchants, economics, technology, engineering and biological sciences (Uchechi, 2013).

The core target of high-quality Mathematics education is the development of problem-solving abilities (Hull, Balka & Miles, 2011). Mathematics skills could effectively passed through concentrated Mathematics instructional teaching strategies to promote learners' retention and understanding. Therefore, in this study, the researcher focused on the effects of teaching through problem-solving strategy on students Mathematics achievement in secondary schools of Murang'a County, Kenya. This study weighed the importance of teaching Mathematics in the context of problem-solving in inquiry-oriented environments characterized by the teacher "helping students to construct mathematical ideas while given profound opportunity in learning process reflected in the classroom" (Lester, Masingila, Mau & Raymond, 1994).

There is a general feeling that to lead a normal life in different countries all over the world, one is required to apply mathematical knowledge. It also believed that greater advancement of technology has an essential background in Mathematics. Therefore, Mathematics considered as a "servant" and "queen" of all sciences. Since all disciplines depend on Mathematics, there is a need to study it. However, Mathematics is part of the education of the life-long learning process, which is necessarily essential in life (Cockcroft, 1982). What students learn in school is not enough to make practical life in Mathematics. They can learn how to arrange working, understanding and apply Mathematical knowledge they acquire in order to deal with complex and diverse questions in their daily life (Singer & Voica, 2013). The learners should have the ability to distinguish how to make use of what they have learned and continue constructing "new information" to solve problems they meet when they complete secondary schools.

Therefore, it is important to use problem–solving strategy to teach Mathematics. The study of Mathematics leads to critical thinking, the correctness of reasoning and improving originality in innovation. Mathematics as a subject has become a necessary tool in all investigations of the world around us. It is cited that representation knowledge of understanding our daily life existing within us help throughout problem-solving (Agwu, 2015). NCTM (2006) proposed that learning Mathematics is essential for students at the secondary school level since it prepares them to do extremely well at higher levels of education. Many countries all over the world continue to have a competitive economy that is a requirement of scientific, technological and social development provided by prospective learning of Mathematics. National Council of Teachers of Mathematics (NCTM, 2012) upheld that every student ought to have maximum equal opportunities to learn Mathematics with the right to deliberate or unintentional bias on tribalism, race, gender, socio-economic status, or language. All students need the opportunity to learn challenging Mathematics from a well-qualified teacher who would make connections to their background, needs, and cultures of all learners to close the achievement gap (NCTM, 2012).

Teaching Mathematics through problem – solving allows the learners to enjoy learning Mathematics amusingly than when it self-generated and enforced by the teacher or textbook (Lang & Evans, 2006). The emphasis of this study investigated the role of problem – solving as an approach, which may improve students' Mathematical achievement. Bruce (2007) who studied ways to get better-achieving students suggested that education experiences in Mathematics would explain that dialogue in Mathematics classrooms is very important in students' achievements. Students live as a community in Mathematics classroom where constructive ideas discussed, developed, questioned and understood in classroom discourse (Bruce, 2007).

According to Singer and Voica (2013), in normal life, people solve common problems to satisfy their various needs. Problem - solving is a long–life process, which practiced in and out of school. The main objective of secondary school education is to equip students with skills for solving problems in a pertaining variety of areas (MoE, 2002). According to Christy and Lima (2007), problem - solving involves higher cognitive skills organised in a systematic order applying an approach of collecting data to make a synthesized informed decision. A supportive teacher who engages students in a classroom environment is important to help in developing students' self-confidence in understanding mathematical concepts (Christy & Lima, 2007).

In Malaysia, students' very low performances at the secondary school level of education continued to decrease tremendously. Comparatively, the Mathematics achievement test conducted by TIMSS in 2012 indicated how well students in Singapore have done. The poor Mathematics performance of the students becomes a major concern among Malaysian educational stakeholders. Malaysian children begin school at the age of six years without considering Piaget's maturation stages. There is no clear policy for promotion and withholding learners, which might affect performance at the secondary level. The performance at the secondary level had been declining. This brought about the need for government to look at the policy for teaching Mathematics. The policy stated that the Mathematics curriculum must focus on the emphasis that relevant information about home, school and classroom context in support of learning and teaching. This policy of Strengthening of Science and Mathematics in Secondary Education (SSMASE) was reform of a noble cause in education to boost teaching and learning Mathematics. This conforms as a key area in relation to Kenya's vision 2030 and global SDG 2050, which stresses mathematical skills to revamp the innovation and growth in economic, industrial, social and political pillars.

In Nigeria, the basic requirement for admission into tertiary institutions is a reasonable achievement in Mathematics at the secondary level (Adeyemi, 2011). Candidates with distinction and good credit grades in five subjects including Mathematics and English language possess the required grades for admissions into Nigerian universities. Emaikwu (2012) in his research, “Effectiveness of three teaching methods in the measurement of students’ performance in Mathematics in Nigerian secondary schools,” there has been a drastic drop in the achievement level of learners in Mathematics for the past decades. Uchechi (2013) proposed that students’ poor performance in Mathematics in Nigerian public examinations is traceable to lack of sufficient content coverage and unexpected teaching methods by teachers. This deprived students of further advancement of technical education and scientifically oriented courses.

Miheso-O’Connor (2009) noted that their teachers at a procedural level, which does not assist students to develop conceptual understanding, engaged students in Kenya in activities of demonstrated algorithms. However, this study involved learners in organized groups to face the challenges of learning Mathematics in secondary schools. Weak students found it significantly difficult, although normal rationalization is usually associated with poor Mathematics abilities. Their difficulties included basic Mathematics facts, reading and interpreting problems.

This study through a quasi-experiment on the effect of using problem - solving strategies addressed the gap that Mathematics achievement progress through learners’ involvement to construct Mathematics, ability to use Mathematics tools and change of attitude towards the formation of Mathematics culture. According to Polya (1957), problem - solving needs practice when deciding on methods required used to solve problems. The first thing to do is to look for hints through guessing and applying experiences similar to existing problems. Hints are the most important skills in solving problems (Polya, 1957). This study investigated how teaching Mathematics through problem – solving in secondary schools in Murang'a County may improve students' achievement.

The study carried out to determine whether using problem–solving approach to learning and teaching Mathematics in secondary schools of Murang'a County has any significant effects on students' achievement. These provided learners with an opportunity to be creative, investigative and explorative of solutions to unfamiliar problems. The learners worked in collaboration with small groups learning through plenty of discussions, solving problems in untried situations, which were encouraged through problem–solving instructional strategy. This study would enable students to become resourceful aligned with the Country's National goal of Education that learners solve problems competently. The intention was to make Kenya middle earning economy with technological innovations **(**Kenya Vision 2030**,** 2010). However, despite this, there was a concern to those interested parties in Murang'a County that secondary school students Mathematics achievements have been on plummeting.

During the past four years, KCSE results in Murang'a County have exposed that about 70% or more students succeeded to obtain grades D, D- and E, which were weak grades as assessed by KNEC. This achievement has been similar to National Mathematics performance according to KNEC results analysis. Nevertheless, Murang'a County becomes an ideal for further interpolation and extrapolation to make the national decision on whether to use problem – solving rather than convention methods. The results of Murang'a County have been on the decline. The numbers of secondary schools in Murang'a County were categorized as A, B, C, and D. These categories of schools based on the past four years in KCSE examinations Mathematics performance prior to the time of this study. Table 1 shows the number of students' Mathematics KCSE entry in each category in secondary schools in Murang'a County from 2014 to 2017.

Table 1: Students in Mathematics KCSE Entry in Secondary Schools of Murang'a County between 2014 and 2017

Category	2014		2015		2016		2017	
Mean score	No of schools	No of students	No of schools	No of students	No of schools	No of students	No of schools	No of students
A:$6 \leq X \leq 12$	20	3220	20	3430	20	3354	20	3697
B:$4 \leq X < 6$	35	3246	35	3241	35	3531	28	3450
C:$3 \leq X < 4$	60	3813	60	4724	60	4879	57	4094
D:$1 \leq X < 3$	195	10327	200	11410	208	11778	228	12301
Totals	**310**	**20536**	**315**	**22845**	**323**	**23542**	**333**	**24633**

Source: Murang'a County Education Office, (2018)

Table 1 shows the mean score which reflects few secondary schools perform extremely well whereas the majority of schools in Murang'a County obtain below-average grades. Further, table 1 also has shown that most secondary schools obtain mean grades below three which proves that majority of students achieve under grade D. These KCSE outcomes have illustrated that more than 70% of students attain grades D+, D, D- and E in Mathematics every year. National performance in secondary schools in Murang'a County in Mathematics within years 2014, 2015, 2016 and 2017 were lower than expected by stakeholders in the County. The category of secondary schools consistently obtaining these low grades in the four years was the same except minimum means changes, which do not place them in different categories. The reason for stratification into four was to consider all students as a cluster of different abilities.

This national examination performance could improve through studying Mathematics using problem–solving situations happening in secondary schools in Murang'a County, whose Mathematics students' mean performance in KCSE were indicated that results during 2014 - 2017 as had been shown in Table 2, with the same trend as shown by results of each category.

Table 2: Students' Mean Performance in Mathematics in KCSE between 2014 and 2017 per Category in Secondary Schools in Murang'a County

Mean scores	2014	2015	2016	2017
A:$6 \leq X \leq 12$	7.284	7.471	6.748	7.418
B:$4 < X < 6$	4.346	4.638	4.184	4.591
C:$3 \leq X < 4$	3.39	3.24	3.044	3.316
D:$1 \leq X < 3$	1.012	1.006	1.068	1.0023
Totals	**4.012**	**3.941**	**3.460**	**3.629**

Source: Murang'a County Education Office, (2018)

Table 2 shows that the means of student's performance is affected by the extreme values revealed that the quality grades A - C+ in Mathematics (15%) which is the entry grades for University admission. These entry grades are important particularly in studying Science - oriented subjects including Business and Economics, (Kenya University & College Central Placement Service (KUCCPS), 2017). The main results make known that most mistakes were owing to misconception, misunderstanding and language interpretation by students. These mistakes pointed more pedagogical approach practiced in teaching and learning Mathematics. Although most teachers' in Murang'a County were trained in Diploma and Bachelor of Education graduates, the strategies they used contributed partially to these declining results. They have a good command of content as well as teaching strategies through training. They have also participated in projects and seminars in the strengthening of Mathematics and sciences in secondary schools (SMASSE). This indicated that the cause of low achievement was due to methodologies employed by teachers. Since the majority of schools' performances was below average, the researcher investigated whether problem – solving approach would alleviate their Mathematics achievement.

Murang'a County results of 2015 per category shown to give a representation of how most students affected by Mathematics performance as they pursue tertiary education. Table 3 shows students' performance grades in Mathematics in KCSE in 2015 in different categories of secondary schools in Murang'a County.

Table 3: Students' Performance Grades in Mathematics in KCSE in 2015 in Secondary Schools in Murang'a County

Category	Entry	A	B	C	D	E
A	3430	768	992	979	653	38
B	3281	90	500	879	1573	240
C	4724	66	317	801	2604	936
D	11410	46	302	742	5399	5136
Total	**22845**	**970**	**2111**	**3401**	**10229**	**6134**

Source: Murang'a County Education Office Mathematics Result Analysis, (2018)

Table 3 has shown that Murang'a County Mathematics performance grades according to ranking, the top 20 schools shown in table 1, seem to have reasonable quality grades particularly the 'A' grades within the County. In these schools, about 79.2% of grades A belongs to category A, 09% from category B, 06.8% and the remaining 05% from category D. These per centages show that for a student in category B, C, and D to obtain 'A' grade probability is 0.05. The general instructional strategies used by secondary schools in Murang'a County investigated were to determine the preferred strategies.

Table 3 also emphasizes that 71.6% of students in secondary schools in Murang'a County obtain grade 'D' and below in their Mathematics curriculum outcome which is below community expectations for career choices.

The most important factors for improving Mathematics performance are students' involvement in creating and thinking through the problem–solving (Polya, 2011). By involvement, it means how much time, energy and effort students dedicated to the learning process through problem – solving, which was the study objective investigated. The main purpose of this study in secondary schools in Murang'a County was to inspire learners to discover, cultivate and apply relevant Mathematics concepts after understanding through problem – solving. Students directed are all the way through the development of inquiry procedure in order to have clue to solve specific problems (Barbeau & Taylor, 2009).

In Kenya, language instruction in learning Mathematics at secondary schools contributed to the low outcome in the Mathematics curriculum. Meaningful interpretation of Mathematics problems students requires learning the mathematical language for comprehending how to carry out operations (Benson & Miheso-O'Connor, 2015). This provided by exposing learners to services of the problem–solving approach. Learners' Mathematics achievement at national examinations caused low retention levels resulting in poor performance. Because of this observation, there was great concern among the educational stakeholders and parents, which is serious apprehension on student future progress. Mathematics is compulsory in Kenya secondary schools' curriculum despite presumed difficulties in learning and teaching Mathematics. The unfortunate curriculum outcome in Mathematics may affect individual future career development at the tertiary level (Ministry of Education, MoE, 2002). Low performance in Mathematics in Murang'a County would affect future generations on career choices. Therefore, because of increasing weak performance in most secondary schools in Mathematics in Murang'a County, there was the need to study the instructional strategies preferred by practicing teachers and the prominence of problem–solving teaching strategy to improve students' achievement in Mathematics.

Secondary schools' students being unsuccessful in Mathematics achievement caused by misuse of instructional strategies in which problem–solving strategy not correctly applied. Through reasonable argument, the question is "Why do incompetent teachers engage learners in problem-solving?" Most teachers argue that they appreciate the breadth of problem-solving activities to prepare students well enough as they connect and support learners. They add that sometimes they have inadequate skills to apply problem - solving. Problem solving as teaching strategy is not been sufficiently articulated in the instructional strategies during teachers' preparation (Martin, 2007). Behaviorist practices emphasize the transmission of knowledge and stress the pedagogical value of formulae, procedures drills and products rather than processes.

1.2 Statement of the Problem

The foundation of all sciences and technology is Mathematics whose functional role affects the society in sciences, technology and business enterprises. The teaching of Mathematics provided a teaching space in which students' contributing factor is central to good performance. Therefore, teaching Mathematics remained a significant challenge, which could address learning through problem – solving to improve students' Mathematics achievement. Secondary schools Mathematics students in Murang'a County had been performing poorly at KCSE examinations. The students' Mathematics results for 2017 has 28.7% of the candidates scored grade C- and above, indicating that over 70% of students scored D+ and below. This shows that most students in Murang’a County will have a challenge in pursuing programs, which are science-oriented at higher education and tertiary level (KCSE Examination Report, 2018).

Mathematics performance in secondary schools in Murang’a County triggered this study to examine and scrutinize the effects of teaching through problem–solving approach on students’ Mathematics achievement. Studies have shown that learners of Mathematics in Murang’a County were not taking responsibility for their work, which is an ideal situation for good performance. The seriousness and urgency of this current situation of continuous decline of students’ Mathematics achievement in secondary schools in Murang’a County prompted this study. There is a need to specify the methods used to teach mathematics in Murang'a County. The students' attitudes towards the subject concerning the problem-solving approach ware investigated to determine their motivation.

This study focused on providing background that engages in an important recreation role to effective teaching of Mathematics through problem–solving approach to improve the achievement of the subject in secondary schools in Murang’a County. This research has shown that learners enhanced their content transfer and improved their achievement through learning through problem - solving.

1.3 Purpose of the Study

The purpose of the study was to examine effects of teaching Mathematics through problem–solving approach, on students’ achievements in public secondary schools in Murang’a County, Kenya.

1.4 Objectives of the Study

The study was guided by the four objectives including To:

(i). Establish the preferred conventional strategies used by Mathematics teachers in public secondary schools of Murang'a County to teach Mathematics over problem–solving strategies;

(ii). Determine students' performance in Mathematics for those taught using conventional strategies and those taught using problem- solving strategy in public secondary schools in Murang'a County;

(iii). Evaluate students' attitudinal change towards Mathematics for those taught Mathematics through problem–solving strategies and those taught using conventional strategies in public secondary schools in Murang'a County; and

(iv). To develop a prototype lesson plan for teaching Mathematics using problem-solving strategy.

1.5 Research Hypotheses

The following null hypotheses formulated to guide the study.

(i). H_{01}: There is no significant correlation between the preferred Mathematics teaching strategies and the problem-solving approach by Mathematics teachers at public secondary schools in Murang'a County.

(ii). H_{02}: There is no statistically significant effect of problem-solving approach on students' Mathematics performance in public secondary schools in Murang'a County.

(iii). H_{03}: There is no significant difference in students' attitude change towards Mathematics for students' taught Mathematics using problem–solving approach and those taught using conventional strategies in public secondary schools in Murang'a County.

1.6 Justification of the Study

According to Kenya Vision 2030, lives transformed through education; therefore, teaching Mathematics through problem - solving must recognized, as it would provide a significant role in education. Problem – solving most likely plays a core driver of achieving sustainable development goals (SDG). Students expected to commit themselves to learn Mathematics through problem - solving as a matter of urgency to a single plan to replenish wretched Mathematics performance grades. For Kenya to achieve vision 2030, Mathematics must been taught through problem – solving where students learn the general ways of solving various problems. When learners use investigations through problem – solving, they learn Mathematics with better understanding, which would help them solve unfamiliar difficulties in the real world.

Mathematics considered significantly important to an individual's everyday life affecting family together with social existence. Individual citizens come across mathematical problems in their daily lives as consumers and workers (Wathall, 2016). Results point out that living in this modern digital world, someone requires advanced mathematical and technical skills in many career successes in achieving necessary tools for the national economy and social life (Wismath, 2014). A problem – solving is a scientific process, which demands analytical and synthetical skills in problem - solving to have critical creative and reflective thinking abilities in which all disciplines must embrace (Posamentier & Krulick, 2009).

The rationale of this study was to prepare students for action in the classroom, give them a better understanding, learning experiences to improve their Mathematics achievement. Problem–solving strategies involve students in exploration and promoting thinking about mathematical concepts rather than telling them key Mathematical ideas.

Students explore problems with their partners or groups and are guided in that exploration by their teacher, communicate their ideas, contribute their insights, apply their previously learned knowledge to new unfamiliar situations, reflect on their experiences and discover new Mathematics ideas (Polya, 2011). When students can express their ideas and represent their thinking both orally and visually when solving problems, they are better to construct deeper mathematical understandings.

According to Vision 2030 where national ethical values for education aims, objectives and philosophy for good governance in a quest for economic, social and political aspirations articulated through problem – solving. This sustains the Kenyan nation through her commitment to democracy and the rule of law by the achievement of national goals of education (Kenya Vision 2030, 2010).

Mathematics knowledge, skills and practices equip learners' minds essentially for effective and satisfying to participate in problem – solving. These strategies become a way of life for students to contribute positively to society's economic, political, industrial and technological development. Mathematics problem – solving is an approach, which serves learners well throughout their lives. Students need active participation in the classroom where they share experiences that help them develop mathematical understanding; learn essential facts, abilities and techniques to assist them in life. If Mathematics problem–solving strategies successfully implemented in classrooms improved for learners' understanding of accountability for consistent, cohesive policies, processes and decision rights of the society as it promotes transparency.

Students find Mathematics to be a difficult and boring subject. This attitude changes to transform the study focus on teaching Mathematics through problem – solving to improve students thinking skills. These further enhanced students' independent mathematical problems solving through minimum assistance. Measuring and developing students' interaction done using Vygotsky social constructivism theory. This theory focuses on teachers and students collaborating through problem - solving to construct Mathematics solutions to problems (Vygotsky, 1978). Therefore, students' guided by the teacher through problem – solving to develop good classroom practices to improve their skills and interactions.

According to Montague (2008), students provided with problem–solving strategies and processes are equipped to make mathematical problems less complicated. The students learn more problem strategies in the classroom community to become better problem solvers. The justification for this study was to impart students' skills, processes and strategies required to become successful mathematical problem solvers. The learners must appreciate and become more confident to solve any Mathematics problem without hindrances.

1.7 Significance of the Study

Researchers have shown that Mathematics achievement in secondary schools in Kenya has been low which attributed to pedagogical skills applied in their classrooms (Miheso-O'Connor, 2009). This study focused on how teachers enhance their pedagogical skills on content delivery through problem - solving. The students adapted learning practices of anticipating, monitoring, selecting, sequencing and connecting, through problem – solving in a classroom environment. These collaborative interactions in small Mathematics communities develop positive Mathematics culture that helps students to improve retention of mathematical concepts and consequently their performance.

Mathematicians in the 21st-century advocate that Mathematics is no longer a telling subject but adapting critical and digital thinking on students for preparation to the problem–solving (Lieberman & Mace, 2008). The study attempted to contribute to the knowledge that learning Mathematics through problem–solving applying exploration, discovery and creative thinking. This strategy can happen in a classroom environment where learners can interact and be actively involved in learning. The research findings aligned with the government of Kenya Education policy and Vision 2030 where appropriate Mathematics instruction and fitting learning in the 21st Century society requirement for engaging learners to the problem–solving competency.

The students' centered learning process where the teacher guides students to the focal point of modern systems of education (Orhan & Ruhan, 2006). The teachers' inputs were how best they develop teaching in order for students to learn and retain Mathematics skills for a reasonable amount of time. The specifics of Mathematics preserved for a long time to achieve better grades. Teaching Mathematics through problem – solving provides the learner with opportunities to develop skills that used in the learning process make decisions regarding various dimensions of knowledge construction. The learners also actively participate in the learning process necessary for improving their achievements. The applicability of the findings of this research was restricted to public secondary school level of education in Murang'a County.

This study of the effects of learning Mathematics through problem - solving enabled both learners and teachers effectively acquire new and relevant practical skills. The learners in particular become innovative and responsible Mathematics problem solvers. Hence, improve their achievement in the subject. This was important to Murang'a County and the Ministry of Education to include problem - solving in Mathematics as it promotes students' confidence to improve their Mathematics achievement through meaningful interactions. It would expected that with the assistance of principals, Quality Assurance officers and Standard directorate in the endeavour to improve Mathematics, achievement and skills in problem – solving within the County would be appreciated. The study might as well help teachers decide on their choice of the appropriate method of teaching and learning Mathematics through deepening their understanding together with educators and policy makers that problem–solving skills in Mathematics has a positive effect on secondary schools Mathematics (Miheso-O'Connor, 2009).

Some researchers have noted that lack of familiarity with word problem structures and language may have contributed to poor students Mathematics performance. This study moderated teaching Mathematics through problem – solving which encourages students' participation in the development of mathematical concepts. This was through the emphasis on Polya's Model relating understanding, devising the plan, carrying out the plan and looking back. For the effective implementation of the problem – solving strategy, teachers should identify quality tasks that make students develop required concepts (Miheso-O'Connor, 2009). The development of these tasks encourages teamwork and collaborations among teachers and students improving to produce activities that promote quality-learning concepts improving students' achievements. When students develop conceptual understanding, they perform better on procedural knowledge.

1.8 Scope of the Study

Murang'a County is one of forty – seven counties in Kenya, where investigation on students' Mathematics achievement in secondary schools conducted. The study covered all public secondary schools, in which stratified according to their KNEC examination previous performance. The participants were Mathematics teachers and students in Form 3. This study used quasi-experimental design. The study investigated the effects of teaching Mathematics through problem–solving strategies on students' Mathematics achievement in secondary school of Murang'a County. The research used Solomon Four Group, factorial design, which required similar conditions for the respondents. This is mainly why one county is important. The school facilities and human resources in almost all public secondary schools were similar.

The choice of the target population as form three was reasonable because they have enough time to remedy their practices in secondary school. The target population is able to understand the questionnaire items and interpret them well. The administrative organizers have a similar calendar of events and geographical climate also the same during the period of the research. The experimental and control groups were exposed to similar facilities. The teachers are involved in the implementation of the curriculum; therefore, expected to benefit from the knowledge, ideas and skills, which would be done through the dissemination of the findings.

1.9. Limitations of the Study

The study restrictions comprised of the following:

i. Students' history and maturation could have had affected the study, but the study took ten weeks which cannot significantly change the results since participants were of the same age.

ii. The teacher respondents answering questionnaires responded by saying 'is' instead of 'should be' which lead to a bias of data on conventional strategies and problem-solving. There was limited time to carry out the research to minister the questionnaire for a second or more times as a requirement of the Delphi technique of data collection. The misunderstanding between the problem-based learning and problem-solving strategies affected the research findings.

iii. The students learning Mathematics was through routine conventional methods where the teacher demonstrate the algorithm for students to follow. Problem–solving approach on students' Mathematics achievement provided opportunities to explore ideas while given chance to extend their creativity.

iv. The effect of reactive or interaction effect on testing of a pretest might increase or decrease a subject's sensitivity or responsiveness to the experimental variable. Controlled by using Solomon's four group designs to improve external validity and use of different schools.

v. Some schools have a high rate of recurrence testing which affects the achievements tests. The researcher had no control over the schools testing policy.

vi. Mathematics taught to students using conventional methods where individuals are encouraged to work themselves. In this study through problem–solving students, construct knowledge through their own experiences. This is the accepted view that student is an active learner promoting retention.

vii. The student has encountered similar exercises in textbooks, in-class algorithms and homework has made learners passive thinkers. Problem–solving strategies help the student to develop general steps that when applied solve problems in any situation. This enhanced through classroom interaction and class communication in problem – solving.

1.9.1 Delimitations of the Study

The study delimited to the following:

i. The study had set the limits of the problem - solving to public secondary schools since they follow the same curriculum and similar environments.

ii. The study was limited to different public secondary schools in the same category. Only one class in form three classes used since the research was experimental.

iii. The researcher considered all public secondary schools in a stratified random sample according to the past four years' KCSE Mathematics performance. The four classes in four schools in each stratum in an experimental group might have affected because most schools had more than one stream.

iv. The study stated a clearly accessible target population of Form three students in public secondary schools in Murang'a County. This was because form one and form two were found to have not adequately covered enough contents in the secondary school curriculum whereas form four was preparing for KCSE examinations.

v. The study delimited to the problem – solving strategy for teaching Mathematics but they used socio-cultural influences to improve student achievement in Mathematics, which had covered in their classrooms.

1.10 Assumptions of the Study

The study conducted on considering the statements:

i. Problem – solving strategy in Mathematics influences student achievement in public secondary schools in Murang'a County.

ii. The facilitators in both experimental and control groups had adequately exposed to problem – solving strategies to learning and teaching Mathematics rather than conventional methods.

iii. Problem – solving strategies have an influence on students' attitude towards Mathematics learning in public secondary schools in Murang'a County.

iv. Problem – solving strategies improve students' performance although they were in comparable abilities and public secondary schools were the same and similar physical environment and performance in Murang'a County.

v. Teachers teaching Mathematics in most public secondary schools in Murang'a County have minimum teaching qualification of Diploma in Education or Bachelor of Education degree.

1.11 Operational Definitions of Key Terms

Achievement: This is a process of measuring rate in performance and change attitude attained by the student in learning Mathematics environment.

Algorithms: Teachers use step – by step procedures that proved to solve a certain type of task in a finite number of steps in conventional strategies, which drive current Kenya's curriculum.

Constructivism: This study allows the learners to do observation and scientific studies geared towards problems through collaboration with others in a mathematical society to create knowledge, done through reflecting on his/her experiences and sharing in a classroom environment.

Heuristic: Heuristic technique or approach anticipated for problem – solving in which learners' study through discovery that employs practical method, but not sufficiently guaranteed to be prime or perfect for immediate goals through which the learners can apply to solve problems. An instructional strategy is an approach customized by a group of learners to achieve learning objectives.

Mathematical problem: A problem is an unfamiliar task without straightaway answer including having no idea of how to do it; the task in which it is not clear to the individuals who are required to take appropriate mathematical action.

Metacognition: Shawn Taylor originally wrote the simplest definition of metacognition in the book *Better Learning through Better Thinking*. This study metacognition means being aware of what you know and what you do not know, understanding what you will need to know for a certain task and having an idea of how to use your current skills to learn what you do not know.

Other Strategies: These strategies include demonstration, discussion, and exposition, guided discovery, collaboration, homework and supervised practice, which when practiced in a mathematical classroom.

Problem–Solving: This is Mathematical skill learning through which students are involved in exploring, developing and applying their understanding of Mathematics concepts. This study will consider teaching students through problem – solving as a means of guiding students through the development of inquiry or problem–solving processes and strategies.

Socratic Dialogue: This is a procedure involving systematic inductive questioning used by guider to lead the learner to Mathematics knowledge through small steps. This study assisted the learner through problem – solving to use inductive method questioning both individually and as a community of learners through breaking down a problem into smaller steps.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This section intends to present a review of literature related to the study including empirical literature where conventional strategies used in teaching Mathematics, problem – solving and students' achievement discussed. Problem – solving and students' attitude including lesson plans for teaching Mathematics. Theoretical literature in which social constructivism, problem–solving and production theories were included. Theoretical framework, conceptual framework, research gaps and summary of literature review.

2.1 Empirical Literature

The part covers the conventional strategies used in presenting Mathematics in classrooms, problem-solving strategy and students' achievement in Mathematics; problem – solving and students' attitude change towards Mathematics. The current lesson plans used for teaching Mathematics discussed in the study.

2.1.1 Conventional Strategies Used in Teaching Mathematics

Mathematics teachers who take care of their students do extra work to create a classroom environment where the classmates form mathematician communities whose activities are to improve learning Mathematics concepts so that their confidence grows vis-à-vis their expectations for high performance. The American National Council of Teachers, (NCTM) advocated that Mathematics teaching through Problem – solving must be the main goal for teaching and learning Mathematics in Agenda for action (NCTM, 1980). Problem – solving plays a vital role to help students develop the skills and functions necessary to solve everyday life problems. The usual conventional instructional models of demonstration of algorithms require the students to follow through increases student dependence on the teacher. Conventional models compared with problem – solving where students develop perseverance independently in solving Mathematics problems.

Research done in America recommended that every person should be taken care of in the learning environment (National Research Council, 1989); therefore, the Curriculum and Quality Evaluation Standards in Mathematics in secondary schools' instruction emphasizes problem – solving as a mode of teaching (NCTM, 2000). The problem – solving encourages students in all aspects of teaching Mathematics to provide them with practice to give them the power to contribute to solutions surrounding them. Similarly, in Britain, the Cockcroft Report (1982), underlined that to improve Mathematics performance, problem - solving should be encouraged given that it offers the learner the opportunity to apply Mathematics logically in inexperienced situations to achieve reasonable progress.

The American and British in their reports observed that through problem–solving, students were able to build, assess and polish their models about Mathematics comparing with other established models. In most countries, different social groups instructors were engaged since the 1980s to edify Mathematics through problem–solving (Brown, 2007), but in Kenya, this did not happen because of Presidential directive in 1981 which directed that modern Mathematics must not be taught to the Kenyan secondary school students (Daily Nation, 1982). Therefore, the new curriculum developed eliminated those topics that taught using investigative methods replacing them with those taught traditionally. This was a setback for problem – solving in Kenya. The curriculum developers continued developing Mathematics where problem – solving applied instead of other methods. Mathematics continued with the opinion that great importance in formal education is significant in cognitive development.

As a result, inadequate material neither researched nor developed, as it happened earlier to teach Mathematics through problem–solving goal of good achievement curtailed. The Kenya National Mathematics Curriculum, developed in 1981, replaced modern Mathematics which was progressive and whose background was to develop Mathematics through learner's experience. The topics such as bases, sets, functions replaced despite that they are basic concepts practiced in teaching computing skills, probability, graphs, sequences, calculus, etc. The new curriculum focused on communicating basic mathematical skills, numeracy and elementary knowledge through topics covering algebra, shapes, numbers, space and data handling. The instructions received at secondary school guides the instructions used in advanced levels and University for high achieving students affects their understanding and use of Mathematics concepts at higher levels.

The educational goals involving problem-solving may not be achieved through teaching Mathematics conventionally since more than seventy percent of students sitting for KCSE have been performing below average in Mathematics. Basic Mathematics offered include topics omitted as part of most courses at tertiary levels. This study investigated the effects of teaching Mathematics through problem–solving strategies to alleviate students' Mathematics achievement at public secondary schools in Murang'a County. This study was intended to improve achievement in Mathematics since it provided students with a good climate to discover concepts themselves; they are capable to stimulate their thinking, reason, connects and reveals as they evaluate the Mathematics encountered in classrooms (Grouws & Cebulla, 2003). The students created relationships by becoming resourceful in developing their mathematical proficiencies and personalities. This strategy was known as teaching Mathematics through problem – solving assessed against improvement in Mathematics in Murang’a County.

Students learn well in a harmonious environment where emotional and social outcomes considered as significant component of educational goals (Pierce, 2005). The teacher creates this by valuing and regarding the students’ Mathematics cultures they bring into the classroom. Teachers must make sure that all their students are involved by avoiding the kind of relationships, which promote dependency through problem–solving (Wang, Tai & Chen, 2010). Teachers are required to encourage students’ relationships in the Mathematics classroom to allow them to think independently, ask questions and proceed to make intellectual judgments. Through these types of relationships, students developed an independent working culture that perhaps is investigative in Murang’a County.

A good teacher often distinguished by her/his ability to choose the right methods and style of teaching practiced in a given situation. The teacher’s performance measured by the students' Mathematics achievements (McDougal & Takahachi, 2014). There are various methods of teaching advocated by different educational thinkers. There are twelve teaching strategies commonly used in teaching Mathematics effectively. First, cooperative strategy in which students in a small heterogeneous group take part in sharing their knowledge (Gelder, 2005). This strategy incorporates different features of effective learning including group interaction, construction activities, positive inter - reliance and individual accountability. Carried out methodology usage in a supportive, collaborative, collegial and interactive classroom environment. Cooperative learning is a social learning method that makes real-life connections through probing questions and scaffolding to make connections to concepts, processes and thoughtful techniques used (Gelder, 2005). The problem - solving needs to allow other conventional methods of teaching rather than considered only one programme among many strategies for teaching Mathematics.

Secondly, there is a demonstration where learning generally directed by the teacher's structural, sequencing and leads the students through an algorithm. This is an effective Mathematics instructional strategy directed through many teaching components including identifying learning goals, organizing and sequencing lessons. The modeling process applied increases understanding strength, providing descriptions and illustrations checking feedback through regular assessment. This improved to support problem – solving as a model for creating a Mathematics community of learners where students share their ideas with instructors and peers (Wismath, 2013).

Thirdly, discussions enable students to develop listening and critical thinking. The students were involved in arguments where students are open to present their opinions on the task given in the classroom (Koray & Azar, 2008). Discussions promote the effective presentation of specific Mathematics concepts, problems or situations guided employing directed questions. The introduction must be interesting, clarification of concepts and information presented in a clear position to previously learned material.

Fourth, guided discovery based on inquiry learning applying appropriate constructivist theories of learning. These theories advocate learners to construct knowledge from their experiences. This process range in various approaches covering fieldwork case studies, investigations, individual and group projects (McDougal & Takahachi, 2014). This is method preferred by many mathematicians and sciences. The student is involved in developing the answer to questions looking for shreds of evidence to support their answers, explaining and justifying them.

Fifth, exposition through research, this method has a low impact on the achievement while compared with other methods of instruction. It is a teacher–centred strategy in which teachers are the major information providers communicating facts and ideas to students (Wismath, 2013). The implication of this method is positive presentation applications relating to new content, which is not available in Mathematics textbooks. The method used for summarizing desperate points of view, focusing on students on critical information.

Sixth, a game in this method encourages students to make up the experiment and practice interpersonal skills in a relatively low environment. Students could assisted through technology to conduct operations simulations to manipulate variables and explore reactions and results (Larson, 2016). Teaching Mathematics through games enhances concept development and algorithms as children love playing so it has a strong motivational effect. This strategy rarely applied in instructing Mathematics since it consumes a lot of time in preparation and application. The game dynamics interplay between the cooperative and competitive learning situations.

Seventh, in homework and guided practice this has been a way of extending learning time in sequencing mastering skills. Designing activities in classroom practice and homework aim at helping students refine and extend their learning. Otten, Cirillo and Held-Eisenmann (2015), explain that Homework can improve achievement in Mathematics when students focus on improving conceptual understanding, flexibility and fluency in Mathematics. The adult involvement should be minimal to help the students do independent practice activities to enhance their learning.

The purposes of homework include reviewing and practicing work done in class by students as they prepare for what learnt in the next class (Koray & Azar, 2008). This improves the student exploration of Mathematics fully when time permits after classroom activities. The learners did not utilize this opportunity when taught using conventional methods because of considering the subject as difficult. If learning resources are available and students consider the benefits of homework done, there is a high likelihood to increase achievement.

Eighth, the project is investigation-based learning which has been a growing method of instruction that addresses core content through rigorous relevant hands-on learning. Projects usually make use of typical structured open-ended questions that force students to construct their solutions through investigation and research. Students using technological tools communicate, collaborate, research and analyze. Students advance their work by creating and publishing their effort for authentic audiences. Projects promote a positive attitude towards Mathematics as student's achievement in secondary school Mathematics improves (Wathall, 2016). Project imports real-life problems into the classroom environment and based on Dewey's philosophy purposeful activities carried in a natural setting.

Ninth, ICT has an impact on modern learning as provided through simulations. Simulations are different ways of representing knowledge by students using better thinking and recalling learnt information (Larson, 2016). Simulations also provide opportunities to envisage possible model whose role-play in dynamic situations. In this manner, promote curiosity, exploration, understanding and problem–solving. Mathematics simulations provide learners with an opportunity to engage in experimental situations, for instance, a rollercoaster design simulator allows students to experiment with a slope, angle and speed. The teacher should be encouraged to be creative and resourceful so that students' Mathematics achievement and attitudes change improved. The researcher intended to regularly assist in manipulating materials to present students lend a hand on experiences that construct useful meanings to mathematical ideas and concepts in the learning process.

The difficulty lies in the times of broadcasts not fitting in with the rest of the school curriculum. This situation may change with the advent of videotape recorders (VTR) and tapes, but such equipment is extremely costly. The viewer can play the programme (or lesson) of his choice by inserting a special cartridge, loaded with a new kind of film, into a special playback unit that sends audio and visual signals into the TV set. The money spent in buying this equipment to offer instruction in Mathematics to make more ways that are effective in delivery except in certain sophisticated institutions. These technological apparatuses used to develop a scientific application in Mathematics teaching strategy aim at improving the students' understanding and achievement in Mathematics (Okigbo, 2008).

A simulation defined as the reconstruction of a situation or a series of events, which may happen in any community. The simulation required that each learner make decisions based on available information and prior training. Simulations are sophisticated games using computers relating to learners' experience and motivation (Larson, 2016). This active approach to learning fosters retention and develops new role functions for both teacher and learner. These are similar to games that are time-consuming, not applicable to all topics and likely to generate a fair amount of noise.

Tenth, question/answer method; this is a classroom environment, where more time and thought devoted by the teacher asking questions than anybody else. One might even say the teacher is a professional question maker (Koray & Azar, 2008). The teacher spends most of his/her time asking questions in class discussions or on assignments and tests. One of the basic principles a teacher stimulates students' thinking to learning was through asking relevant questions. According to Angier & Povey, asking questions and studying given answers enable a teacher to measure and evaluate student's thinking in learning progress. Teachers regularly stimulate four main types of thinking activities by asking questions (Angier & Povey, 2010). These types include remembering, reasoning, evaluating, or judging and creative thinking. The major limitation of this method was that it requires time to think and answer questions. The teacher asks probing questions, which guide learners to select correct answers carefully. This method observed commonly spread over in secondary school teachers' in Murang'a County without involving the learners.

Eleventh, supervised practice; is self-study on the part of the student but in the presence and under the direct supervision of the teacher. Supervised practice conducted in the regular periods or case of residential school in the after-school hours. It also carried out in the reading room of the library where a teacher may put on duty to watch and guide students. Supervised practice is a well-known feature of boarding secondary schools (Highly rated on this study) where all students are to assemble at their classrooms at night to study under the supervision of a teacher.

The supervised study creates a formal and studious atmosphere for the self-study of the students. The teachers' presence makes the atmosphere more disciplined and congenial for hard work. The students are in the company of their classmates, which makes mutual collaboration possible. In Mathematics, a student can benefit a lot from supervised study. Even if he/she is a day scholar, he/she may be required to study for some time after the school hour for supervised study. This period utilized for disposing of the homework provided in class. If the school provides some extra time for supervised practice, students may not have to carry any burden of homework with them.

Twelfth, Method supported through Plato's dialogue relating to the Socratic technique, which challenges students to think analytically and critically is the problem – solving. Mathematics teachers carefully ask the question to give student guidance to be resourceful. This is a cooperative learning strategy developed by David and Roger Johnson to structure and focus on interactions in the classroom. The students initially work individually, then in pairs and finally as a group continue to increase discussion involving the whole class to arrive at a consensus.

The foundation in collaborative learning underpinned by the theory of social constructivism whose knowledge supports collaborative activities mostly based on four principles (Kay, 2010). One student according to constructivist is the primary focus of instruction viewed as an active learner. Then followed by two students' interaction and doing as a most important step in collaboration. The students work as individuals, then in pairs and small groups before being engaged to discuss the concepts in a classroom community.

The learners enabled to take their own responsibility for discovering, reviewing, organizing and consolidating existing knowledge. The third phase was working in groups as an important mode of learning in the relationship. Finally, structured problem approaches developing solutions to real-world Mathematics problems incorporated into learning progress. The basic understanding of Mathematics structures in filling spaces, finding additional sequential meanings and formulating Mathematics knowledge into new conceptual frameworks (Clarke, Goos, & Morony, 2007).

2.1.2 Problem–solving Strategy and Students' Achievement in Mathematics

Effective teachers were responsible for their students' effort to work individually and collaboratively to make sense of their thoughts. Research show that it is challenging to comprehend new concepts or solve problems when diverted by the views of others (Dursun & Dede, 2004). The teachers will not act as model mathematicians for mathematical practices for students to imitate but work with them. Teachers must ensure that all students given equal opportunities available involving them to create, think and work independently in a mathematical environment. The learners were not always required to obtain varied solutions but also conflicting perspectives, which need further investigations (Pratt, 2006).

According to Burns and Myhill (2004), the most important human resource for developing mathematical patterns in cognitive thinking are teachers; therefore, they bring about management, facilitation and monitoring of their students' participation and record solution, emphasizing efficient ways of resolving problems. The teacher invites each student to clarify her/his solution to others ensuring that discussion retains its focus, aim and generating Mathematics knowledge. Teachers encourage students to pay attention, participate and value one another's opinion. Students also expected to evaluate different points of view as they engage in healthy arguments to show their perspectives and because of their action as well as accepting others' strong insights (Wenger, 1998). Each student viewpoint is a justification in the development of resourceful feature Mathematics information.

The study shows that when individuals work as pairs of partners, then small collaborative groups can facilitate students' interaction as a living Mathematics society of learners. These engagements provide practical and emotional support to students' needs. The learners are involved in identifying the nature of a task and make out possible techniques forward (Voica & Singer, 2013). The groups modified to work as pairs in smaller groups, which are useful for enhancing personal commitment effort working with others. Higher-level thinking boosted through making possible exchange and testing of ideas. When small supportive groups are used, students learn how to make their conjunctures, take part in mathematical arguments and validate their answers considering the knowledge gained from the class.

Anderson and White (2004) distinguish the role of constructivist educators who are teachers should not take the position of the "learned on the stage." Instead, they proceed to be “the leaders on the side" students are given openings to examine their Mathematics adequacy presenting their considerations. The effect of constructivism theory on the educator considers knowledge and experiences learners bring to the classroom and contribution among peers. The process of problem – solving helps students in constructing non – routine solutions through the exploration of various problems. They develop the processes to look at the reason, generalize and conceptualize as they solve unacquainted tasks using a collection of strategies to draw valid conclusions. The learners reconstruct their information through the active process of inquiry. They find out what had facilitated so long as there are necessary resources (Norton, McRobbie, & Cooper, 2002). This knowledge actively constructed presented as the dynamic discovery of their ideas. It done by arranging concepts with the assistance of assimilation of new and old knowledge.

Constructivist advocates that learning programme was a long line of student’s active inquiry to build important information presented in reasonable order. This is an appropriate and flexible permit development as a process of discovery, which helped their teachers to assimilate old and new thoughts. The student is required to interpret the real-world problem in different ways. However, the teacher would create circumstances where students search and reflect on their processes without interference (Wismath, 2014). The learner contextualizes learning through problem – solving to improve students' Mathematics achievement; in a favourable environment and supported by cooperative learning to construct knowledge. They are encouraged in the development of partial hypotheses providing scaffolding at the right time and level to contribute ideas. Finally, but not least, learners were provided with opportunities to interact with one another sharing information, for example, those who have done less practice will learn from those who are high scoring as they contribute to each other's input to be in the process of discovery (Hattie, 2009).

Hattie (2009) supports that learner's think mathematically means developing mathematical attitude valuing processes of the problem – solving in which abstraction of having prediction how to apply them. This included developing competence by providing necessary help of those tools used in the service and anchoring support in problem-solving strategies in real-life situations (Schoenfeld, 1995). Mathematics as a subject seeks to understand patterns that permeate the real-life world and mind within students' communities.

According to common people, Mathematics establishes rules that learned makes it easy, but this is not the case by the community of mathematicians. Therefore, Mathematics taught through problem - solving facilitates learners to develop mathematical language to help them generalize Mathematics patterns in the real world. This study through problem–solving enabled students to determine the heuristic approach commonly used in teaching Mathematics in secondary schools. It will assess the students' move beyond rules and direct things in the language of Mathematics (Marzano, Frontier & Livingstone, 2011).

The purpose of creating a learning community whose aim is to share in making Mathematics rational through problem - solving. The model of schoolroom community is significant to learning through problem - solving since it facilitates active participation in social interaction, as mutual support to one another. In the classroom community, the teacher supports students to acquire knowledge from each other through corroboration (Wismath, 2016). The structures of the classroom community diverge with openings for students to work independently, assortment teams, small groups, as well as a whole class.

The teacher as a facilitator regularly continues demonstrating and accepting students' ideas. The students' way of thinking and asking questions, making conjectures, and validating their solutions will be encouraged (NCTM, 2000). John Dewey (1954) wrote that democracy should be "the idea of Community life itself" (www.ascd.org.publication, 2014). Dewey considered school or classroom as a community of learners, parents and teachers who should live in harmony and collaboration. He had the notion of creating classrooms communities living, working and developing skills in sharing problem – solving. Dewey believed that democracy in the classroom community is not just a place to live but also an active contribution in the process of sharing a sense of purposeful vital learning from each other through participation in the learning activities. This includes asking questions, listening to one another's standpoints, comparing thoughts and imagining alternatives (Dewey, 1954).

A classroom community becomes an important force in learning Mathematics in four ways; discussion of meaningful content, classroom relationships through playing games, building connections and problem–solving. Mathematics teachers through the determination of planning systematically creating important tasks required to involve the students in the process of the problem–solving (Coltman, Petyaeva & Anghileri, 2006). The students' involvement in practical knowledge development always emphasized in the Mathematics classroom so that the values of learning Mathematics are appreciated (Wathall, 2016).

Development of practical skills through involving students' ideas takes place when they are actively engaged in learning. They are involved in high-order thinking in the Mathematics classroom community as they focus on Mathematics problem – solving on Polya's (1957). Polya based the problem–solving strategies into four important steps in the thinking process, work out a plan for the answer, implementing the strategy and looks back to the solution. According to Polya, there is no structured process in solving problems. The procedure of solving mathematical problems is to build from various intellectual processes, which practical usage and rationalized in getting an acceptable solution. Polya continues to articulate that multiple attempts to obtain solutions started from the improved initial attempt. Students considered capable to apply heuristics approach learning and creatively mature with mathematical ideas for solving either general or specific assignment of mathematical problems (Wathall, 2016).

Teaching students to use the heuristic approach is not an easy task, but the study investigated the successful method employed on heuristics approach learning to consider Polya's four steps. These steps include understanding the problem, devising a plan, carrying out the plan and looking back (Polya 1957). Students' who strongly believe that applying the heuristic approach shows good abilities in their discovery method in mathematical problem - solving. The problem–solving strategies documented to help teachers in Murang’a County comprehensively meet higher-level learning standards for students, which would improve their performance. Ravitz and Blazervski (2010), contend that problem–solving approach centres students’ experiences to the investigation of problems in which students produce high-quality products or presentations demonstrating their learning.

Mathematics education in the 21st century is facing world real-live problems significantly, nurturing creative thinking and cultivating productive ways of learning through problem – solving. The educators have discovered the importance of metacognition to attempt to modernize Mathematics teaching and learning. The preparation for the new generation requirement on 21st Century in Mathematics needs students’ full participation in problem – solving. People imagine that Mathematics is always one of the difficult subjects in school. This picture of Mathematics affects students' understanding of anticipating problems. Therefore, students must be actively involved in constructing Mathematics through problem – solving to improve their achievement.

Von Glaserfeld (1995) contributed to education that: Educators have noticed that many students were quite able to learn the necessary formulas and applying them to the limited range of textbooks without examining situations. Students are faced with different problems; they demonstrate that they might not have understood relationships in relevant Mathematics concepts and conceptual relations advanced by problem – solving. The students should overcome the conceptual deficit through problem–solving approach where learners investigate mathematical ideas through sharing, inquiring and discussing among themselves (Grouws and Cebulla, 2003). There is reasonable improvement in Mathematics achievement.

Accordingly, to Schoenfeld, learning to think mathematically implies developing a point of view, valuing Mathematics processes through problem – solving. There is a conceptual generalization with intention of prediction to apply these concepts in different circumstances. The emerging capabilities using those tools of the problem–solving to build in the service aim of accepting structure of intellectualizing solutions in Mathematics makes problem–solving a reality (Schoenfeld, 1995). The learners develop skills that help in understanding, reasoning and better achievement outcomes through problem – solving.

NCTM (1989) observed that the minimum academic achievement in Mathematics in secondary schools is through competencies in reading, writing and arithmetic which all students expected to achieve. The more advanced academic achievement will require training in problem - solving where teachers reserve for the selected few students with high intellectual capabilities. During this industrial age, an educational system that can meet the population economic needs of society today requires the students to be well equipped in problem - solving. The most likely occurrence to motivate the recent momentum of curriculum development problem - solving should centered on Mathematics. Therefore, Problem–solving referred to as a method enhanced by the teacher as a facilitator or guide to the learner to develop the perception and concept formation through task solutions (Orhan & Ruhan, 2006).

This study investigated the action taken by teachers to improve the students’ perception and concept formation through problem - solving. The learners took an active role in the learning process, according to their own needs and pace. The nature of the subject demands original reasoning in nature and manipulation of acquired previous learning experiences. The organization of Mathematics concepts promotes understanding where given opportunities make reasonable decisions. A variety of dimensions in the learning process regarding self- actualization achievement in Mathematics accomplished through studying problem – solving.

The channel of instruction in which Mathematics content achieved through problem – solving. Koray and Azar (2008) posit that effective teachers maintain to pose problems from the beginning to the end of delivery of Mathematics content. Students answer these problems since they know it is the only direct means of self-fulfillment apart from solving the problems (Koray & Azar, 2008). They engage students in problem–solving means exciting students to investigate Mathematics concepts to expand and apply their understanding of these concepts (Leiken & Rota, 2006). Mathematics experience viewed as a difficult subject, but to overcome this notion of a complete misunderstanding, problem - solving becomes a process that can extricate itself from this problem.

The process of obtaining the solution to problems with which problem – solver is not acquainted is referred to as problem - solving in Mathematics. The unfamiliar procedure applied by problem - solver through a strategy of action of selecting appropriate method intended to accomplish specific resolution. Often a comprehensive answer is resolved when the problem solution seeker has tried several times to attain the correct attempt. Failure to gain an amicable solution using various procedures enables the problem – solver to consults other members of the community. If the Polya model is applied the problem – solver will sequentially oscillate forward and backward within the four stages to produce an agreeable answer following provided hints. Watson (2002) provides problem–solving model including managerial processes, which other instructors such as Schoenfeld and Flavell considered as metacognition. This study investigated mathematical abilities of the learner improved through problem–solving approach in Mathematics. The students' mathematical thinking and ability to comprehend enhanced by using Polya's fourfold problem-solving process.

Mathematics is an important subject whose aims, objectives, purposes, and rationales will not exist in a vacuum but considered to a bridge in problem – solving. Mathematics belongs to people. Those individuals or social groups that is why it taught as problem – solving. Teaching Mathematics considered a widespread and highly organized social activity. Social constructivism agrees with the possibility of divergent multiple aims and goals among different persons (Trochim & Donnelly, 2006). Generally, Mathematics social activity in society ultimately intents, goals, reasons, and underlying principles that achieved through problem – solving. Students were prepared to function in such a society while teachers are ready to take responsibility to promote in their classrooms Mathematics cultures in experiences and the acquisition of problem-solving strategies (Ali, Akhter, Hakimabad, & Khan 2010).

There are differences between problem–solving approach and other teaching conventional strategies. Problem–solving method involve searching non - routine questions, using wide range of methodologies of solving unacquainted tasks in everyday life relating Mathematics. It applies different methods to develop new problems through analysing, reasoning, generalising and abstracting to discover mathematical concepts (Anderson & White, 2004). Problem – solving is a teaching Mathematics approach, which contributes, to students' development of abilities provided with opportunities to ask questions, explore new ideas, while the teachers themselves are guiders. Teachers become good listeners and observers apart from exerting their authorities and answer givers regarding Mathematics (Norton, McRobbie, & Cooper, 2002).

Kenya secondary school syllabus (MoE, 2002), regard as the importance of problem – solving in Mathematics as a working Mathematical component. Student practices strategies of applying questioning, communicating, reasoning and reflecting as critical aspects of Mathematics learning method (Clarke, Goos, & Morony, 2007). Most teachers particularly in Murang'a County are not able to interpret syllabus in context of problem – solving. Students are required to explain problems frequently while giving them opportunities to deal with complex questions and reflect on their thinking as an important part of problem – solving. Students make conjectures and justify their answers through collaborations with other students in Mathematics community. The communication occurring throughout the process of trying to find solution to problems help all students to understand the problem indifferent perspectives. This unlocks ideas to great number of plans for getting at least to reasonable solution to the problem (Singer, Pelezer & Voica, 2011).

Research dealing with teaching methods by Johnson and Johnson (2007), found that, student-centred instruction intended to motivate students to form closer association with one another in working together in classroom community. This was in line with our national goals of education to promote national unity, foster patriotism and justice, which completed through problem–solving (Kenya vision 2030, 2010). Student-centred group does not only score higher academically, but also gain some social skills through cooperative work creating a Mathematics culture in problem – solving. In contrast, teacher-centred lessons geared towards lecture and individual work of the student delivered less prominence in Mathematics teaching. The recent developed syllabuses encourage problem – solving and support teachers to use it.

Researchers suggest that many teachers do not practice Problem–solving strategies because it consumes time despite its prominence (Anderson, Sullivan, & White, 2004). Anderson and White (2004) articulated that many teachers do not distinguish between problem – solving and problem - based Mathematics teaching strategies. The process of exploring non-routine questions in order to obtain solutions which is communicated to others whereas problem – based learning is where specific tasks are provided for learners solve.

The problem–solving approach used to develop knowledge through process of analyzing, synthesis, generalizes and making reasonable conclusions emphasized during problem–solving accomplishments. This study incorporated conventional teaching strategies and Problem – solving approaches into the lesson in Algebra and other topics and document the changing nature of the tasks devised in discovery method. Most Mathematics problems classified as transformation problems as in attendance effectively change connections in problem – solving (Heason, 2004). There are three stages of problem – solving. The first step known as initial state where problem solver will study the problem origin. The second goal state that when problem objectives are clearly stated. The third stage is legal operators connecting problems to be solved using search techniques to obtain mathematical solution.

The procedures used to solve problems include means – end analysis engaging student attempt to reduce original problem into smaller tasks (Christy, & Lima, 2007). Then, the differences occur when each small problem flanked by state encountered towards achievement of the set target state using necessary operators. Those who were familiar procedure use strategies that well known while novices' problem solvers are different since they use trial an error. The novices require more practice to be able to apply means – end analysis working backward from the goals by setting sub-goals (Christy, & Lima, 2007). The novices later acquire experiences to use reverse procedure they followed in forward working sequence. The experts can choose appropriate relationships, which show the way to goal state immediately. Using previous experiences to identify the moves they can make appropriately recognizes each problem state.

Realistic problem state configurations depend on basic principles leading to expert – novice distinctions on memory. This might become instrumental in determining mode of problem categorization in problem–solving strategies. Experts' schemas allow distinguishing between problem states and identifying course of action. These associated by moves undertaken to categorize problems according to schemas. Novices navigate through alternatives for correct surface structures when classifying problem. These are main factors of differentiating experts and novices in Mathematics problem–solving skills (Wismath, 2013).

There are a number of positive benefits expected when teaching Mathematics through problem – solving, which urges teachers to include it in their Mathematics programme. These include making Mathematics interesting and enjoyable method of learning. The students consider learning new Mathematics concepts through problem – solving as it makes available greater understanding and produces positive attitudes towards Mathematics. Wismath and Good (2014) advocate continuation helping teachers to engage students learn thinking, creativity and flexibility through problem – solving making them junior investigators in Mathematics. Generally, problem – solving skills encourage students to develop cooperative skills, familiarize themselves with similar approaches to other subjects taught and create better citizens in secondary schools who consequently become members of the society. Teaching Mathematics through problem –solving reinforces discovery of new knowledge to help student meet everyday challenges.

Mathematics skills learned improve logical reasoning where individuals can participate optimally in Mathematics society. The process of just genaralising the rule to follow in Mathematics to obtain correct answer does not help. They need to be able to choose logical process whose deductions that develop their own algorithmic rules. These reasoning developed during problem – solving can be valuable skills. Problem – solving is line of attack in thinking rather than just as the means to an end of obtaining right answer. The Mathematics learning through problem – solving gives an inspiration of this study. The investigations show a significant improvement in Mathematics students' achievement in performance and choosing problem state that is likely to connect with problem at hand (Myers & Torrace, 2003). The teacher's work is to pose questions, which lead students to make a move that shows they have sufficient understanding before solving them. These movements allow students to discover the mathematical ideas that are uncovered in a problem and come up with more than one solution through teacher guidance. They can work as a community and reflect on the problem as they work together.

2.1.3 Problem–solving and Students Attitude Change towards Mathematics

The students attend school from different backgrounds with differing needs that derived from home environments background, religion and languages. They possess varied capabilities and perspectives creating Mathematics culture (Kranda, 2008). The teacher is a single human resource who takes an important role in developing students' mathematical identities. Students' positive attitude towards Mathematics, manipulated by teachers using their experiences through problem – solving. Students' performance raised by positive attitude rather than their perspective comfort level that Mathematics is difficult subject. It also enables students to gain greater confidence as well as their capacity to learn Mathematics making sense in problem – solving. The students take responsibility for their own learning as they are encouraged by their teacher's giving guidance. In the classroom, students simplify their challenges through interaction with each other and their teacher (Watson & Geest, 2005). Those challenging tasks that require students to think deeply for Mathematical ideas, which they connect with confidence rather than depending on their teacher answers. These opportunities help students to find Mathematics enjoyable and relevant to their attitudinal change.

Classroom traditions play vital position in developing students' mathematical way of thinking and acceptable judgment of their ideas. Students' everyday exercises engaging them to contribute solutions in a mathematical enquiry promotes problem–solving activities in cooperation. According to Angier and Povey (Angier & Povey, 2010), teachers need to clarify their expectations about how their students should contribute and participate in the development of their concepts and how others might respond. Development of students' mathematical proficiency guided by teachers showing interest in their learners' ideas. Teachers help students to express and develop those skills necessary for solving problems (Singer &Voica, 2008). The modeling of ideas through practice and evaluating their application are encouraged to discuss with others in order to make mathematical judgments using suggestions voiced by their classmates.

Teachers promote students' positive attitude towards learning Mathematics by creating conducive environment and encouraging community action through collaboration in classroom dialogue. This information shows widely acceptable relationship between classroom situations to improve academic achievement in Mathematics through effective student's incentive in learning Mathematics. Researchers have shown that students' inspiration is highly related to Mathematics achievement in learning environment (Cheng, 1994; Uguroglu & Walberg, 1986). Colakoglu & Akdemir (2008), enthusiasm is closely related to Mathematics achievement which may be the cause of decreasing trend in Mathematics performance in Murang'a County. The decline in Mathematics performance may be associated with students' interest.

Researchers suggest that effective Mathematics learning depends on environment and caring classroom Mathematics community who interact freely (Henningsen and Stein, 1997). According to Forman (2003), the key areas for successful way of creating an environment in Mathematics problem – solving is classroom culture. These loving classroom communities are ready to assist each other in problem – solving. They develop helpful relationships among teachers and students. These relationships will promote learners' contribution in creating classroom norms, making decisions and setting objectives. There are responsibilities and clear expectations through opportunities for collaborative learning. Students' necessity to provide with adequate time for completing tasks through discussions. They offered enough time to work on tasks, which are not restricted to makes problem–solving approach interesting and meaningful activities.

In Mathematics classrooms, there are many factors, which influence students' positive attitude, given that studies show that students who appreciate their classmates have high prospects, hopes of being friendly with strong relationships in their classroom community (Scherer & Steinbring, 2006). Several educators have confidence that students learn Mathematics by having information communicated to them and left alone to generate their own knowledge through construction of their solutions. Constructivists such as Piaget, Bruner and Vygotsky, suggest that some people learn better if they are able to construct their own Mathematics rather than listening to lectures or reading textbooks.

In this study, problem – solving was used where learners had opportunities to talk about their ideas and create solutions. A procedure of construction method used where foundation knowledge of Mathematics concept is operationalized is called problem – solving. The learner is actively involved in creating her\his individual gain knowledge and understanding by connecting what communicated with prior data and experiences (Miller, 2008). Mathematics teaching and learning acquired when students assemble information and understanding in social interactive process through problem – solving. These students gain from each other the spirit of sharing their points of view, thoughts; ask questions and constructing on their own methods and thinking using others' contribution.

The process of communicating about their rational working operation to create essential ways of thinking needed in learning procedure is encouraged (Bruner, 1996). The key factor in guiding students toward interactive and constructive discovery learning in classroom environment. The necessary features to create such an environment include small group discussions; student willingness to produce problems investigated solutions. The dynamic involvement to students emphasizes on reasoning, substantiation and personal interpretations rather than drawing attention to correct results only. Most research papers underscore that classroom activities provides good chances for students' active contribution in cooperative learning in creating and maintaining safe Mathematics classroom environment. The main objective of instructing Mathematics through problem–solving giving students in favourable conditions, which support their change of attitudes and subsequent improving their achievement fundamental gap of current study. They can as well build positive emotions and intrinsic motivation, which are vital success in being able to be creative and have better conceptual understanding (Csikszentmihalyi & Schiefele, 1995).

Kenya Education national goals are promoted through problem – solving comprise of Nationalism, Patriotism and National unity. Problem–solving doings contribute to promotion of national goals of education (MoE, 2002). Study investigated whether caring Mathematics community learners can be able to live interact and socialize as Kenyan's citizens. Problem – solving can improve this community of the youth to acquire sense of nationhood. The extended use of problem–solving support removing conflicts and promoting positive attitudes of mutual respect to each other through classroom interaction (Scherer & Steinbring, 2006). Recent development in education learning trends focused on the teacher rather than the learner. Teachers spend more time in assessment rather than syllabus coverage.

2.1.4 Mathematics Lesson plans used in Secondary Schools

A lesson plan is the teachers' road map of what is going to happen in the classroom it includes main objectives, sub-objectives, learners' activities, teachers' activities for guiding learners and resources used in the classroom to advance problem - solving. Mathematics considered as difficult subject by most students, but it depends on the lesson plans made since it made in an enjoyable way to create students' interest. First, the teacher needs to identify the learning objectives before the class meeting. The teacher should determine what students want to learn and able to do at the end of the class. According to Markova (2012), to specify good objectives for student learning, the teacher should answer some questions such as what the topic of the lesson is. What does the teacher want students to learn and take away from this particular lesson? Although it is important even to manage time, most teachers did not plan, and others have common lesson plans from a common pool in the Murang'a County.

Second, a teacher should develop an introduction where the activities designed to help the learners to understand and apply what learned before the commencement of the current lesson. The introduction developed to trigger the students' interest and personal experiences with the resources already familiar to them. Asking questions to develop current lesson, does not usually happen as teachers ask questions on the previous lesson even when they do not relate to the current topic. The sequence of lesson development introduced where students' process skills integrated by identifying types of relevant knowledge to apply in an unfamiliar situation.

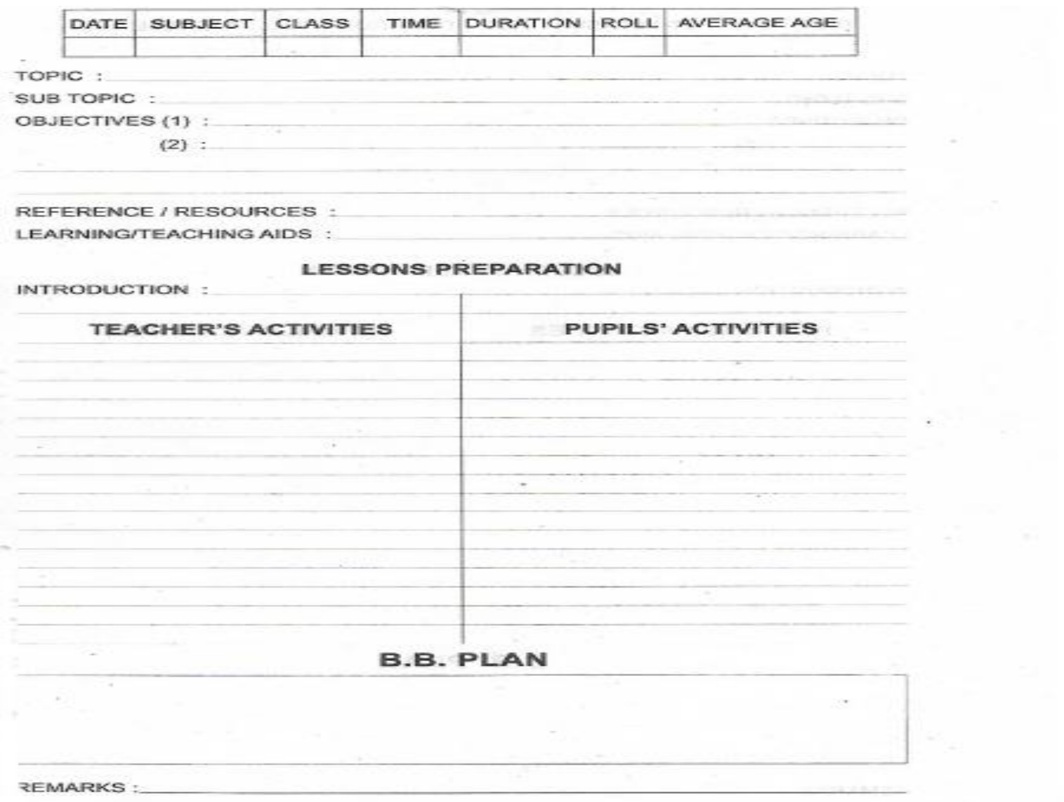
Third, the main body of the lesson plan made to capture the students' attention through usage of different learning styles. The time spent on each activity estimated includes identifying strategies check the understanding of the concept. The following could assist in designing learning activities in the lesson development. These are what the teacher will use to explain and illustrate the topic in different ways to engage the students. (http://www.crlt.umich.edu/gsis, downloaded 4 – 03 -2019). These questions are teacher–centred approach rather than student–centred approach such that what would students need to do to help them understand the topic better. The teacher would decide on what kinds of questions to produce that meant for discussion and ensuring student understanding.

Fourth, the lesson plan also checks for understanding and develops a conclusion through going over the material covered. When planning a lesson, it is important to think about specific questions to check students' understanding of the Mathematics concept. Students’ answers reviewed measure understanding of a topical idea. There is a need to conclude the lesson by summarizing the main points and connect different impressions within a real-life problem. At any rate, most teachers use commercial lesson plan books shown which instead of having learning activities and resources have teacher’s activities transforming lessons to be more teachers – centered. The study developed problem–solving prototype lesson plan to assist teachers to use it to enhance the mathematical approach to improve achievement.

A good lesson plan contains relevant information, which has all structures and components of teaching. This relevant information gained helps future planning, but the methodology used should be one to determine student achievement. The contemporary Murang’a County students’ performance does not have evidence of a correlation between life achievements.

Figure 1, represent commonly used lesson plans which are usually bought from bookshops for commercial purposes. The lesson plan decision-making outcomes adventure in secondary school teaching. They are not arbitrary writings to convince self or others that they are to diagnose learners’ needs, set objectives, set out learning activities and predict evaluation. Commercial lesson plan teachers’ usually use stress on teaching activities conflicting answering important questions; what to teach? How it is going taught? Also, at what level is the student going to be instructed?

Prepare Local Lesson Plan



DATE	SUBJECT	CLASS	TIME	DURATION	ROLL	AVERAGE AGE

TOPIC :

SUB TOPIC :

OBJECTIVES (1) :

(2) :

REFERENCE / RESOURCES :

LEARNING/TEACHING AIDS :

LESSONS PREPARATION

INTRODUCTION :

TEACHER'S ACTIVITIES	PUPILS' ACTIVITIES

B.B. PLAN

REMARKS :

Figure 1: Lesson Plan

Source: Commercial Lesson Plan, 2018

The commercial lesson plan shows school, class, subject, teacher, dates and time. The question, who is teaching promoting direct answer,? What, where to whom? Objectives, syllabus elaboration, presentation sequence, methodology and resources. What is learned is split into two including teacher activities and learners' activities, which show that a demonstration method assumed teachers use in teaching Mathematics. Step-by-step procedure for students to follow hinders conceptual development. This discloses why the student performance had been declining.

2.2 Theoretical Literature

Problem–solving strategy as a goal of learning Mathematics had been justified by some educational theories learning. These contribute to improving students' achievement in

Mathematics. This study used three main theoretical literature reviews social constructivism, problem–solving and production theory. Constructivism carefully planned to be a philosophical viewpoint about the nature of facts, which are an epistemological bearing as advocated by Piaget, Bruner and Vygotsky (Mcleod, 2015). Problem – solving is measured to be higher-order cognitive process and intellectual function. These theories were improved by the Polya model in problem - solving. The production theory relates social constructivism and problem – solving to make the required achievement on students.

2.2.1 Theory of Social Constructivism

There are three most important schools of thought advocated by constructivist theorists in education. These are constructivist learning theory, constructivist-teaching theory and social constructivist theory. This study adapted social constructivism's new approach in education that claims humans better enabled to learn and understand the information as they participate in its construction. The social constructivism theory had developed and supported by many educational thinkers like John Dewey and Kenneth Gergen. Underlying principle knowledge in which knowledge constructed through the social interaction of learners to gain possession of good results in social processes denote social constructivism (Gergen, 1995).

The social constructivism model formulated from social realistic consensus and based on social communication of students. Formulation of social constructivism theory procedure in this study developed by three eminent persons, namely, Jean Piaget, Jerome Bruner and Lev Vygotsky. Piaget reflected social constructivism as a cognitive learning theory that focuses on distinct mental processes of constructing the meaning of mathematical concepts depending on maturation. Students individually construct mathematical patterns according to their reasoning and experiences hinged on their growth. The knowledge building from experiences in terms of existing information schemas requires social interaction of the learner. This shows the way of action taken personally and socially depending on class environment setting advances problem–solving abilities. Therefore, problem – solving as social constructivism in Mathematics learning, relates learner has known patterns to new concepts through interface connections (Herbert & Carpenter, 1992).

Learners have a common preposition that the classroom environment must assist learners to think creatively and imagine. Their differences are in terms of discourse carried out in these classrooms. These classrooms are having more than one component of methods that assist students to construct new knowledge. The new information acquired enable student to think and reflect on Mathematics concepts and models. The teacher who successfully applies these methods of social constructivism encourages students. They can easily reevaluate their methods and reconstruct their ideas in classroom management to build classroom cultures. Social studies educators must always recollected that the purpose of social constructivism in Mathematics is an effective way of creating citizenship while these methods help in acquiring more Mathematics concepts "behind the scenes" time rather than "conventional" methods. Teaching Mathematics through problem – solving improves students' achievement in classroom interaction.

Vygotsky's socio-cultural theory describes that human learning as a social process helping young mathematicians to apply human intelligence in Mathematics cultural society (Vygotsky, 1978). In a constructivist classroom, teachers' centredness focus on learning replaced by the support of students' centredness relationships. The teacher is no longer, an "expert" who pours knowledge into passive students in the classroom but the director allowing students' contribution to move to learning progress. Students, however, should not wait as if empty vessels designated in filling without any input. According to Vygotsky's constructivist model, students have a strong desire actively include, in the development of the process of Mathematics learning through problem–solving (Doolittle & Hicks, 2001). Learning becomes an interesting venture for a student acquiring wonderful outcomes. This in turn causes students to change their attitude towards Mathematics. Students become more attentive, eager; think independently to discover Mathematics themselves through Mathematics constructivism (Kaur & Yeap, 2009).

A constructivist classroom environment is a place where both teachers and students contribute to knowledge development. Problem – solving in Mathematics causes dynamic of ever-changing real worldview to develop the ability to solve problems successfully. These will stretch inactive individual learners who wait drilled and memorize facts to active participants in learning. To execute the mandate of the problem – solving, it takes to consider what students currently believe and experience with correct or incorrect solutions. The special consideration for this question allows the student to make direct and decisive independent decisions on Mathematics learning which bears fruitful consequences on Mathematics achievement (Acosta –Tello, 2010).

Despite having the same classroom environment, each student will gain experiences to improve understanding in personal perspectives. The students successfully learn the Mathematics aspect of solving the problem and learn to think mathematically (Maschkovich, 2012). Understanding construction means a course through which learning involves active and continuous engagement. This underwrites changing of conceptual comprehension where learner focuses on their increase in achievements (McConnell, 2008). Research shows collaborative action influences how students construct new knowledge although they believed that could reject after testing (Vula & Berdynaj, 2011). Learning is not a passive process but active participation, by students taking a dependable part in learning themselves.

There are similarities between constructivist and social constructivists in classroom discourse, but later extend learning beyond constructivism to social interaction. It emphasizes the value of cultural education through the creation of collaborations and the culture of young mathematicians. Vygotsky advocates a culture in which students develop cognitive tools needed for Mathematics problem - solving. Teachers and parents within learner's environment are means by which Mathematics culture transmitted. These include mathematics language and social-cultural context. The electronic forms of accessing Mathematics information recently had considered (Norton, 2009). The social constructivists encourage peer interaction as a collaborative learning process mediated and structured by the Mathematics teacher.

In conclusion, social constructivism theory suggests how students develop and acquire mathematical concepts through social interaction in problem – solving in Mathematics. The main principle of this theory is the development of realistic knowledge based upon social consensus in the Mathematics community. This theory through problem–solving challenges conventional models whose common theme of Mathematics and some available learning programmes do not involve Mathematics communities. The conventional model emphasizes on memorization of facts, drill instead of the development and generalization of Mathematics concepts. There is a need to reform the way students taught Mathematics by using social constructivist theory. The social constructivist theory plays a crucial role in the approach. Mathematics content communicated to students. Social constructivism influences problem – solving that causes changes occurring in the teaching of Mathematics in classroom cultural communities where the interaction of the learners and mentors is paramount.

The teacher's role is to change in classroom environment modify their traditional exposition methods to adopt 21st-century Mathematics learning through problem - solving. A key aspect in the strategy of problem – solving in Mathematics was to start by breaking down material into a simpler fashion to build up complex problems. Social interaction and multiple perspectives become a driving strength between the peers relating to present situations in the modern world. The teacher's responsibility turns out a guider and facilitator in a Mathematics classroom environment.

The classroom environment creates a "learning community" whose main input is a culture where students interact as active participants in creating and thinking together. For the teachers to accommodate these changes of a social interaction-learning environment in a classroom, they are required well versed in the theory of social constructivism as applied in problem - solving and understand its implications (Brown, 2007). Theory of social constructivism clearly understood by Mathematics teachers, applications should effectively cast off by students in problem – solving in Mathematics.

The fundamental importance of problem - solving is the most prominent orientation among Mathematics teachers and educators. There is uncertainty that constructivism moment the dominant ideological and philosophical position, which this study used interchangeably. Mathematics taught through problem-solving changes the functions of a teacher to a director and facilitator whereas the student fosters a positive attitude towards the problem – solving as well as Mathematics. The students exposed; experienced to share learning effectively and formalize Mathematics. When the problem–solving approach applied in teaching Mathematics, it develops a repertoire-acquiring strategy that compliantly when faced new with situations.

Model for cooperative and collaborative learning as teaching strategy involves the learner in a process of socialization, external collaboration, combining ideas and internalizing concepts through sharing knowledge between individuals. The externalization by interpreting information to transformation into comprehensible forms that understood by others and involving combination by synthesizing knowledge through fitting the elements together to form required through social constructivism in problem–solving Mathematics.

According to Fosnot (2014), in the constructivist-learning environment students expected to play a more active role in building and accepting to be answerable to their learning. When learner ownership of process problem – solving in finding solution need to be given support. The solution obtained may be inadequate; the Mathematics teacher becomes an active participant learning process. The students’ role remains as active contributors to their education as they accommodate and embrace new Mathematics ideas for better understanding. One of the main students' control is constructing Mathematics through a learning process. Students reflecting on experiences gained in the course of problem-solving relearn what they had conceived earlier. Most students are reluctant to give up their pre–conceived representation of ideas. They may usually from time to time reject new thoughts without evidence on the basis, which they previously considered true knowledge (Mascolol & Fisher, 2005 downloaded: www.ucdoer.ie, 2016). Sometimes students are not aware of why they hold strongly certain schemata. Mathematics students need to test thinking skills, and actions through relevant strategies in problem – solving. However, they distinguish what is right to adjust the direction of their Mathematics thinking through problem–solving approach.

Fosnot argues that knowledge is a collective responsibility where learners deserve access to understanding through interaction with different communities. This might influence their thinking in a reasonable direction (Fosnot, 2014). The problem–solving strategy employed prescribed as the Socratic Method. In this method, the teacher expected to have the right answer, but the student's task is to predict the answer. When the deduction is not correct, the teacher actively pursues it through logical questioning intending to capture the required response. Student learning concept needs to receive different ways of perceiving new things to develop mathematically. Vygotsky (1978), social constructivism for the teacher is to offer guidance to learners through the zone of proximal development while their peers play a vital role in learning how to solve problems.

The primary argument by NCTM was that Mathematics teaching through problem – solving, as an attempt to achieve better grades at secondary school as understanding Mathematics becomes part of the students' performance (NCTM, 2000). Teachers continue to say that executing certain procedures does or does not ensure that students understanding the process of problem - solving. Mathematics taught through problem - solving maintains that it is a precise way to build strong young mathematicians.

This philosophy of allowing learners to investigate Mathematics through the discovery method is the desire to change the teacher role as the only source of useful information. Teachers should know when to provide the right information at the right time to their students. Students are supposed to originate, develop or discover social conventions in solving the problem on their own. The teacher provides alternative solutions as the students come up with further improvement of the present answer. The teacher as one of the Mathematics classroom community learners may point out ideas that embedded in the students' work to produce suitable solutions in Mathematics problems (Lijjedahl, 2005 downloaded: www.peterliljedahl.com, 2017).

Thinking mathematically is an intended goal in problem – solving which involves practice and reflection. There is an argument that when students given the time needed to engage and reflect on the problem, they have a real and quality understanding of the problem – solving. The teacher expected to make possible a supportive environment that helps students gain confidence in problem – solving in Mathematics. Students can be able to construct their answers for better achievement through reflection on their successes. The experiences that teachers have in implementing problem – solving in supportive environment classrooms through enquiring, thought provoking, and insightful in a culture of thinking should be encouraged in Mathematics teaching (Yackel & Rasmussen, 2002).

There are structures, which will allow immediate move toward cognitive development in problem-solving goal. Schemas are structures defining cognition, as a schema is a well-defined cognitive structure, which allows solving problems. Schemas require a large related cognitive process to attain their function. The student involved in solving the problem must concentrate on the differences between the current problem and goal states. The student using a complex problem–solving strategy may be to obtain the final solution, such process called means-ends analysis. Mean – ends analysis comprises student expectations that may be able to see the end before the beginning. This enables her/him to determine the best strategy to apply in order to attain his/her reasonable goal.

The cognitive load imposed on the learner as means-ends analysis interferes with learning during problem – solving which may not solve the problem since it is an important factor for a student to explore (Goldstein, 2011). Teachers may be perhaps acquainted with learners' cognitive development of mathematical concepts. These help learners to grow and develop patterns of learning which vary from one individual to another. The teacher regularly assesses individual students or groups to measure their performance according to the task. Thereafter, design and modify instruction to meet learners' needs in each area of development in problem - solving. The social environment, which is critically develop helps that individual learner to understand as well as develop the body of knowledge (Goldstein, 2011).

Piaget's work based on students' cognitive functions regarded by many educationists as the founding principles of constructivist theory, study research development on similar principles. The effort observed that learning occurs through adaptation to community interactions within the environment to develop Mathematics culture. Piaget argued that the existing cognitive structures of a learner determine how new information changes their standpoint in self - governing environment (Moschkovich, 2012). Therefore, the learner was expected to be actively involved as a contributor in constructing his\her own knowledge through problem - solving. Piaget further observed that students assimilate new information applying existing mental structures; formation of concepts gained in complexity, power, and their understanding in existing situations as they grow in richness and depth in cognitive concept development. This thinking is a core concept of the constructivism view of the learning process (Moschkovich, 2012).

Development of learning communities encompassing students, teachers and experts who are engaged in authentic tasks context closely related to work done in the real world depend on a conducive constructivism environment. Learners provided with multiple perspectives through which discussion or debate enables them to see issues and problems from different points of view. Further negotiations interrogating meaning develop shared understandings with others in the community.

Students applying personal Mathematics understanding to solve problems central to objective of teaching Mathematics in secondary schools' level using problem–solving in training individual appreciation. Conceptual development reflected enabled students to analyze and synthesize Mathematics concepts to improve their skills (Umay, 2007). The study examined that problem–solving expertise as of utmost importance enable students to improve Mathematics achievement in secondary schools. Problem–solving abilities awarded priority in Mathematics teaching produces a better outcome. The research carried out on determining factors on how problem-solving skills acquired has been inadequate; therefore, this study attempted to reasonably answer this important question both theoretically and practically in Murang'a County. Most theoreticians assumed that to understand the problem–solving process is through a practical solving large number of conventional problems to gain necessary practice on the problem – solving. Although the main theoretical framework based on social constructivism, the study adapted Polya,'s conceptual framework concerning problem – solving.

In summary Vygotsky, 1978 emphasized complex Mathematics knowledge and skills learnt through social interaction in problem–solving activities. Social collaboration provides students with the opportunity to use others as resources sharing their ideas with others and become active participants in the joint construction of knowledge. These experiences help the student to learn how to communicate their ideas through the development of public thinking and guided to sound successful Mathematics problem solvers. Teachers' accomplishment to encouraging their students to evaluate their work, forming conjunctures, and publicly sharing mathematical ideas successfully makes them better mathematics achievers.

2.2.2 Theory of Problem Solving

Historically resolving problematic circumstances had been the heart of educational objectives, which parents and teachers expected to achieve from their students as they shape capability in problem – solving for future generations (Saeed, Shahvarani & Behzadi, 2012). In the contemporary educational systems, teachers imagined facilitating the process of learning where students are responsible and self-directed to solve Mathematics problems. Students have reproduced standard solutions or techniques provided by their textbooks. Minimum time dedicated to teaching learners how to carry out the investigative process. In 1980, problem–solving was principally considered an important goal for mathematical education, research was undertaken with an aim to understanding the nature of the problem–solving through creating instructional programs that can help students' develop knowledge-based, heuristics and nature, that they could be able to explain the process of the problem–solving more effectively (NCTM, 1980).

Under modern Mathematics, students subjected to the right environment develop investigative skills through consultative interaction, where they were engaged in creating, thinking, and writing Mathematics in problem – solving through the discovery method (Towers, 2010). According to Polya (2011), Mathematics is insightful, helps in understanding, reasoning and solving complex mathematical concepts through powerful problem–solving heuristics (Polya, 2011). Generally, experienced educationalists use an analytical method as an instructional tool to enhance learning when making

Mathematics relevant and practical (Cox, 2011). While students are involved in problem-solving skills, they promote their independent learning. Mathematics instructors believe that the quintessence of learning Mathematics is a way of implementation of skills in testing all aspects of problem – solving, exploration, examination and building conjectures. Learners can construct, build problems from assumed situations to create new problems through modifying conditions specified in the problem.

Historically, John Dewey was an American Philosopher in the early 20th century who viewed education as schooling employed in the critical investigation of problems and problem–solving (Dewey, 1916). Dewey further referred problem – solving as reflective thinking. Further, Mathematics believed as a process of connecting thinking critically about learning how to solve problems. Discovery learning perceived as an important strategy fostering problem–solving skills that emphasize education objectives achievement. The learner develops in considering different learning environments, abilities, content and instructional practice adapted through guided experiences (Bruner, 1996).

Bruner suggests that constructivist theory effective Mathematics learning should be structured concepts through problem–solving skills. Bruner's theory of constructivism development proposes three modes including language, learning and discovery (Mcleod, 2015). Bruner argues that language includes primary means of communication to convey Mathematical information. Language also is important for increased ability to deal with concepts understanding and organization. When language absorbed show that an individual freely deals with the appearance of complex problems construction. According to Bruner (1961), the rationale of education is not to pass on knowledge rather the application of ideas through seeking a solution (Bruner, 1996). Education remains a process to facilitate students to think independently in problem-solving dexterity.

When faced with an unfamiliar situation the progression follows a pattern composed of three phase's enactive, iconic and symbolic representation. According to Bruner (1996), enactive stage supports that student begins to develop understanding through active treatment of Mathematics. This manipulation promotes students' opportunity to play with mathematical concepts as they move iconic stage. During the iconic period, students are capable of making visual intellectual descriptions using available materials. Through encouragement, students can use abstract ideas to represent real-world circumstances in a given environment. In this symbolic phase, students can assess, judge and reflect critically on Mathematical concepts. Bruner suggests that a true instructional designer; works with the learner to unpack information so that the student is capable of learning it with minimum instructions (Mcleod, 2015). This belief sharply contrasts with Piaget's and other constructivist theorists who believe that certain maturation should attained.

Language learned Bruner explained that it creates possible techniques to communicate concepts information in a spiral curriculum. This involves sequencing complex ideas to the existence of structured simplified levels within the given environment. Usually, educational goal viewed as intellectual development of the students occurs where conditions manipulated in their favour. Ideally, learning should be processes leading to learners to able to solve problems by themselves. Bruner recommends that learners' construct their knowledge through organizing and categorizing information using a coding system through discovery learning (Bruner, 1996). The teacher's starring role is the facilitation of the learning process employing designing lessons that help the student discover the relationships between bits of facts acquired.

Unfortunately, the claim that students must wait to qualify when guider considers student have reached an appropriate state of cognitive maturity does not support constructivism theory. Bruner adopts a different view in which deliberated that learner capability of appreciating complex ideas at any age. Bruner (1960) conflicted with Piaget's conception of readiness and contended that schools waste time trying to match the complexity of Mathematics material to student's cognitive stage of development. Learners held back by curriculum developers who insist that certain topics deemed difficult for the student to understand. This means transfers of materials covered at a given level curriculum structured to include those taught previously.

Participation in learning through the discovery method presents a fundamental aspect of Bruner's principle. He disagrees totally with students learning mathematical concept knowledge without thinking like mathematicians (Bruner, 1996). For instance, thinking in the symbolic stage but relapsing at times when he\she aided to obtain her\his solutions through logical growth. Incidentally, expected that older students with the advanced mathematical ability to solve problems in a way discovered by mathematicians solving similar problems. Although among Bruner's three stages of development were completely inconsequential. Bruner emphasized culture where Mathematical notions are structured such that their views and others around the world in which they live.

In conclusion, Bruner expanded his theoretical framework to include social and cultural learning aspects. This was encompassed using three principles namely that learner must be instructed when ready to apply experiences and contexts making student able and willing to learn. Secondly, the curriculum needs prearranged and designated in a spiral organization such that students can easily grasp materials learnt. Thirdly, instruction is planned to facilitate the learner to go beyond the information given and extrapolate in process of construction (Bruner, 1996).

Problem–solving being a critical component of comprehensive 21st Century Mathematics education since it invokes key skills in the present day's students as critical thinkers and problem solvers to achieve SDG (Lawson, 2016). Mathematics forms the basis of decision-making in many disciplines in the long-life learning process. Therefore, learning and facilitation in Mathematics remain in heart of education problem–solving strategies. Learning Mathematics aims to link school to everyday life-giving skills required for resolving problems. Students acquired the necessary techniques for the workforce needed in fostering Mathematical thinking (Suurtamm, Quigley, & Lazarus, 2015). Mathematics involves learning how to solve problems. The mathematical investigation and presenting concepts in good communication of discovered concepts and ideas making connections to the daily basis on contemporary life.

There are serious concerns that Kenyan students continue to have low Mathematics achievement in KCSE examinations (KNEC, 2018). Polya's theory (Polya, in Watson, 2002) defined problem - solving as a dynamic process involving several activities: understanding the problem, making a plan, carrying out a plan and looking back to the Mathematics problem tackled. The definition was applied to the mathematical discussion in this review rather than problem - solving. Problem – solving involves systematic and thoughtful strategy gathering information to make an educated decision.

Higher-order philosophical thinking would support or change mathematical ideas depend on the problem.

Problem–solving model generally defines a problem through understanding it, developing a plan through familiar situations, implementing the plan by carrying out the process, and evaluating the progress. The problem and its context are analyzed from range being simple to complex, depending on data specified. The model generally applied intrinsically to reflecting commitment, assessment and data-driven decision making as a systematic process. The specific tasks information will go through the process of finding, using, applying, and evaluating them. The practical applications of Mathematics make it an essential discipline to the individual and society. This aspect of Mathematics developed through problem–solving strategy on students' Mathematics achievement. This provides students a vehicle to construct their mathematical thoughts as they take full responsibility for their learning information brought by the curriculum spiral approach (Taplin, 1995).

What were some best practices for instructional Mathematics? Generally, the best practice is an action taken as a way of producing the desired Mathematics achievements. Typically, people think that the best practice is a strategy organized in a lesson to promote a deep understanding of student ideas in Mathematics. The following list recognized has been an instructional strategy, which considered best practices in Mathematics teaching. The instructional strategies identified must ensure that activities have taken well thought to be a learner – centred with emphasis on inquiry in problem – solving in Mathematics.

In the first phase, the student attempts to understand the problem through asking questions fluently in his\her own words. The student will reframe the problem in his\her own words and pointing out the unknown, given data and condition (Singer et al., 2011). Do you identify what is given and the goal? Is there enough evidence? Is there a similar problem you had solved? When applicable, the student will then illustrate the problem using a diagram and point out unknown conditions, and underlying statistics. According to Polya, problem – solving the first phase involves interrelated applicable complementary processes of creativity and critical thinking (Polya, 1957). This will engage advanced intelligent abilities including both the construction and generation of ideas. The processes involved use experiences in problem – solving to evaluate problem solutions.

The second phase is devising the plan, in which the student is required to choose the applicable strategy best learned and gained by solving several problems (Singer et al., 2011). You will find appropriate connections between the unknown statistics. They may ask themselves whether they have seen a similar problem before. If a similar problem had been seen with a slightly different form or if the student relates to a known problem. Do they know the related problem, which they may use directly or indirectly in reasoning? Solving current problem student have considered auxiliary problems before immediately makes the connection on the way found, whereas not obliged to make trial solutions. If you obtain similar problems, you will eventually plan the solution accordingly. If no direct answers are joining found solution, between what is given and the problem, then the teacher asks leading questions to help students capture a bright hint (Polya, 1957). These hints prepare students fairly to make the problem clearer.

The third phase is when the students implement the proposed plan using the strategies that she\he has suggested consequently solving the problem (Singer et al., 2011). The process is carried out until a new course of action is suggested when a reasonable solution is not obtained. The student attempt to solve a given problem enough amount of time to obtain a sensible solution. If she\he is not successful, she\he will seek further hints from others in their classroom community. Otherwise, she\he shelve the problem aside for a while. The teacher should ensure that student checks each step. During this phase, the student accepts responsibility and ownership of their accountability decisions. Mathematics learners through problem - solving improve their ideas to become a life – long accomplishing professional growth and development in obtaining the correct solution. The teacher ensures that students carry out the plan encouraging checking each step.

The main reason for learning Mathematics is to become better problem solvers empowering all aspects of life. The student asks him/herself exactly what type of information provided is adequate to determine all the answers required by the question in given conditions (Polya, 1957). Since when learner clearly understands the answers to those two questions, learners are then ready to devise the plan and execute obtain solutions.

Finally, the fourth phase will be looking back where the student examines the solution obtained and reflects on it (Singer et.al. 2011). Do answers obtained satisfy the statement of the problem? The solution attained can use data at glance in different problems. Can the approach apply to some other problem? Can a student competently extend his\her solution to a more general case? In this final phase, the teacher encourages the student to reflect on their results and find out whether they can improve them differently (Polya, 2011). This process of reflecting is by reading the problem carefully repeatedly; decide whether the method used to solve the problem has been successful. Then, check on the work done to make responses create sense through problem – solving so that the same term used could explain their answer better (Polya, 2004).

Teaching begins with the learner, environment and classroom community. This study provided each student with new knowledge, skills to interpret information and make reasonable decisions through problem - solving in Mathematics. Teachers must develop the learning of their students by creating enabling environment where their students enjoy developing mathematical patterns, which vary among individuals bringing out unique individual differences to the learning process (Crebert, Patrick, Cragnolini, Smith, & Webb, 2011). The learners need a supportive classroom environment where learning thrives as they participate in the development of Mathematics culture.

Effective teachers have high expectations of each student's achievement when they implement appropriate developmental challenges. The learning experiences within a variety of learning environments, helping each learner to reach high standards in their potential growth. The study intended to apply standards using problem-solving strategies blended with other conventional methods. There was a combination of strategies, which were the basis of the development of knowledge. These include understanding of how cognitive aspects, etymological, social, emotional and physical. This requires learners' recognition that as individuals who bring different personalities and family backgrounds to a Mathematics classroom. Problem–solving skills developed through abilities, perspectives, talents and interests in mathematical community culture and surroundings (Crebert, et al., 2011).

Teaching Mathematics through problem - solving ensures consistency where everyone needs to know what method of solving a problem is appropriate in life. The process of problem - solving follows a scientific learning procedure in which individual biases are susceptible and perceptions are clarifications of answers. The management of the learning process goes after six steps model that provides the focus for the group and helps set the agenda of problem – solving. The students using individual approaches work on the model given using their suggestions all at the same time then later combine their ideas.

A particular method analyses data to make decisions based on problem – solving easier for collaborating groups to reach consensus. The strategy, which is crucial to yield better result since ideas were tested eliminating those which could not work in their particular situation. The problem–solving model makes use of series of rational stipulated steps to help the group identify the greatest sources for providing the best solution. Then, the group considers all options everyone must build a convincing case by considering all possible solutions provided by their members. The answers presented interpreted to process a useful methodology development. The substantial rationale of action presenting conclusions of procedure improvement activities in Mathematics cognitive concept development.

Figure 2, Shows problem–solving improvement cycle which helped to change a more convincing development process. Steps sequentially arranged in a closed curve emphasizing cyclical, continuous nature problem – solving adopted by learners and teachers to obtain the best results. The systematic approach allowing decision-making based on data, rather than guesses defining the source. Determining the root causes of a problem is a positive process of the problem–solving success. These avoid learners' reactions to superficial indications of problems while devising permanent solutions.

Problem–solving procedures followed by quick delivery of fixes rather than relying on presumptions (education.nsula.edu, 2016).

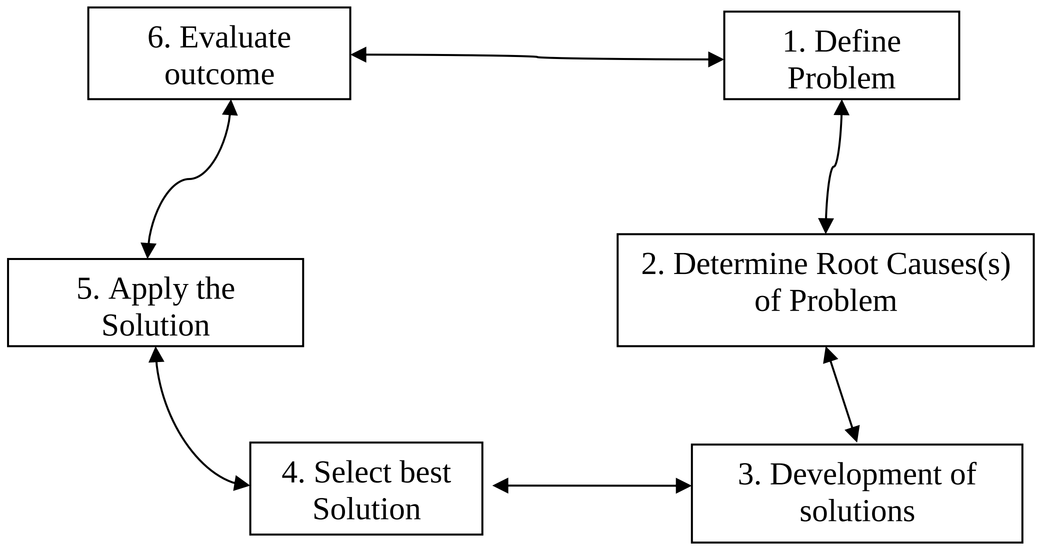


Figure 2: Problem-Solving Cycle

Source: Researcher Improved Polya Problem Solving Cycle

Figure 2 demonstrates that the first step requires understanding the problem defining it properly and determine available resources to achieve the necessary goal. The next step determines an action plan, which provokes thinking activities to direct problem – solving. The application of strategies settled on during class discourse is the process called mathematical modeling. Carefully think about the possible consequences of carrying out the action plan. If the problem has been solved cautious analysis of the steps carried out as discussion in groups and whole class. The last step evaluates the solutions by reflecting on what you needed to learn by solving the problem.

2.2.3 Theory of Production

The theory of production deals with principles of an economic process and production by which inputs transformed into outputs. The production uses inputs resources include capital, labour, raw materials, land and time connected to outputs. Education occupies an important position in every major economy of the world in the investment of human resource. This production process connects significant input of good services to output problem solvers in the economy. Education production is a function involving the relationship between school student inputs and measure of school output on sensible achievement. This study representing the educational production process on Mathematics where students taught using problem – solving are weighed against those students taught using conventional methods (Demie & Lewis, 2010).

The vital Mathematics achievement on students' performance and attitude as human development value measured by Mathematics score at KCSE. Education is a production function process that takes a moment of time and space since taken as a flow of concept production measured as a rate of output. Three aspects of the Mathematics production process namely: the quantity of the service input; a form of the service created and temporal service produced.

Mathematics as an education production function is a process of combining various services inputs and plans how would be intervened to make creative and resourceful individuals. The study carefully analyzed theoretical perspectives and empirical results on learning Mathematics through problem – solving regarded to performance. Learning took place to improve students' Mathematics achievements for reasonable output. Mathematics learning changes conventional methods to the problem–solving approach. Learning takes place as students can use any piece of knowledge, draw and justify their ideas (Demie & Lewis, 2010). This way they feel more convinced in a cultured classroom community. Therefore, the learning environment provides a natural setting for students, to present their various discoveries to their groups or completely class while teaching through problem – solving. Mathematics best studied through social interactions reaching shared understanding comprised best output.

This study concentrated on the relationship between school inputs as students and facilities. The conventional measures of school outputs on students' achievement scores on Mathematics related to inputs and benefits. An explanation of relationships between scholastic inputs and Mathematics achievement output, on one hand to economic performance on the other hand. The systematic relationship, exploration and resource between scholastic achievement and economic performance are considered. However, present results based on the educational Mathematics evaluation of the schools' outputs. The facilities in the school create reasonable media for students to enable to extract main information from the problem. Teacher instructional disposition continues to assist the student-learning disposition. Problem–solving approach as intervention where students' individually work in one of the factors influencing implementation. Students' collaborative works on the problem within time constraints help the learner to have a positive interaction.

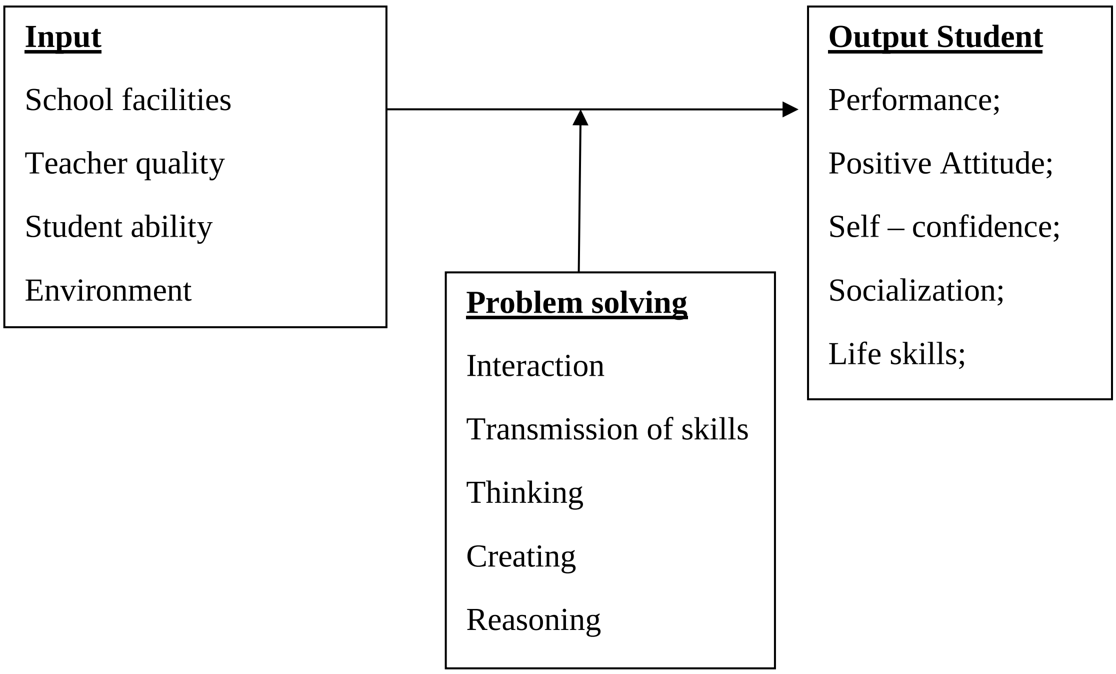


Figure 3: Production on Student Achievement in Mathematics

Source: Researcher (2018)

Figure 3 demonstrates that curriculum in Mathematics processed through problem – solving in some classroom interactions, to improve students' achievement in both performance and attitude. This is a production theory since the input the student scored better grades, which help her/him to continue with education at tertiary level in science-oriented courses. The production model reveals that common inputs are things like teacher quality, school resources and family attributes. The outcome is student achievement in varied dimensions.

Riasat, Hukamdad, Akher and Khan (2010) in their paper on the "effects" of the problem – solving recommended that problem - solving improves students' achievement in Mathematics. Resnick (1987) argued that using problem–solving approach, people develop the practical ability to apply problem – solving when technology fails. This emphasis changes from teaching problem-based to problem - solving where students guided to develop individual skills (Lester, Masingila, Mau, Lambdin, Dos Santon & Raymond, 1994). There has been an effort to clarify the meaning of teaching Mathematics through problem–solving approach. Current investigations emphasize inquiry-based learning environments where the teacher characterized as a facilitator for students. Mathematical ideas and processes occurring in students were through the construction of questions asked to deepen understanding by the teacher. This method guides students in developing mathematical procedures and activities to solve problems. The student is responsible for designing the cause of action, which assists in creating, conjecturing, exploring, testing and verifying mathematical topics through contextualized problem–solving (Lester et al., 1994).

Principally interactions between teacher-student, student-student and student-teacher in Mathematics dialogue and agreement between parties are consented to embrace the problem–solving approach (Artzt, Armour–Thomas & Curcio, 2008). Students' interaction in a manner that backs and challenges one another is premeditated thinking contributes to the problem – solving. Teachers provided with enough information to form background intending to give hint to the problem, while students left to clarify, interpret and struggle to construct more than one solution (Wood, Cobb & Yackel, 1992). The teacher's role was to help students bring possible generalizations to light and draw attention to important points raised.

Teachers accommodating right or wrong answers in a non – evaluative way from students (Shellard & Moyer, 2002), should allow themselves to guide and coach learners. Students asking insightful questions and sharing ideas profit fully in the process of solving problems. Teachers developing questions provide appropriate intervention while they step back letting students make their own informed decision on conducted individually (Prontheroe, 2007). The learners' conceptual achievements, understanding, develop practical literacy as well as promoting strategic competence by meaningful problem–solving involvement to obtain better production.

Schoenfeld (Schoenfeld, 1995) suggested that good quality input improves problem - solving that extended to lead exploration and generalization in Mathematics. Despite the input, the approach ameliorates omitting good output depending on schools and families. First, mathematization described as a process, which valued highly in abstraction and having a preference to apply in Mathematics activities. Second, proficiencies developed become tools of the trade that gives service goal in understanding structure making mathematical intelligence (Schoenfeld, 1995). Problem solving generally regarded as the most important cognitive activity in any professional context.

Problem-solving is an everyday single vehicle that seems to accomplish Mathematics achievement to higher levels with all three values of Mathematics listed as functional, logical and aesthetic (Burns & Myhill,2004). However, learning Mathematics through problem - solving was considered a requirement in formal educational settings, since the understanding of mathematical processes usually limited the policy environment. Research on instruction and theory in problem-solving Mathematics has had little attention paid in the Kenyan curriculum. The study assessed conventional instructional methods generally used alongside problem – solving on teaching Mathematics to realize quality outcomes in secondary schools in Murang'a County.

Problem - solving plays a prominent role in Mathematics education in secondary schools since students learn to be independent and cooperative. Teachers knowing problem – solving meaningfully incorporate in Mathematics teaching yields good student achievements. The undertaking is not necessarily an easy task since Mathematics teachers insist that they cannot complete the syllabus on time. Mathematical activities include potential accountability to intellectual challenges in cognitive development. Students' mathematical thought and development submitted enhances problem – solving which is essential (Cockcroft, 1982). The key central theme in learning and development of Mathematics comprises social, the collaborative activity of experts as they guide in problem – solving in Mathematics (Burns, 2004).

The Mathematics curriculum in Kenya should take problem – solving as its focus in the process of helping students achieve their expectations. Mathematics goals outlined in the curriculum will have problems pitched at an appropriate level of education. This research concentrated on the objectives that relate Mathematics to the real world, helping students to become more confident in the subject and improve Mathematics engagement. The problems in the real world provided a challenge to students through whom they expected to develop mathematical knowledge necessary to give them meaning to skills and concepts used. Students need to feel that they have a reasonable chance of solving problems, either by themselves or in groups (Burns & Myhill, 2004). Mathematics is a complex cognitive activity, which demands students to reason, communicate thoughts, make relations, apply knowledge, and offer services to particular areas, which is simplified through problem-solving. Literature on Mathematics education describes problem - solving by way of separate performance number of activities as mentioned. This includes doing word problems, generating patterns, and mounting geometric constructions, understanding figures, and proving related theorems (Watson, 2002).

In conclusion, consideration of the impact of school resources focused on a factor of production has a statistical difference on effect on outcomes. Production theory indicates that possessions never influence achievement without intervention with the problem–solving method. Accumulated facts on education function stipulate a clear relationship between resources and students' outcomes. The quality of teachers through their training does make a consistent difference when assessed against student achievement gains.

2.3 Theoretical Framework

The empirical and theoretical literature reviews make available taxonomy of some areas that originate syntheses of ideas of research (Stoilescu & McDugall, 2010). The social constructivism theory of learning through collaboration and seeks to describe how group dynamics improve social and personal competencies. This would evolve out of the social environment within where learning could occur without hindrances. Social constructivism and problem-solving theories describe the mathematical process through which concepts are developed. Students improve their Mathematics achievement while working in a good environment, where social interactions help them to communicated Mathematics ideas. The mathematical process-taking place in a good environment, helps concept development improves Mathematics achievement on students aligned to educational goals and policy.

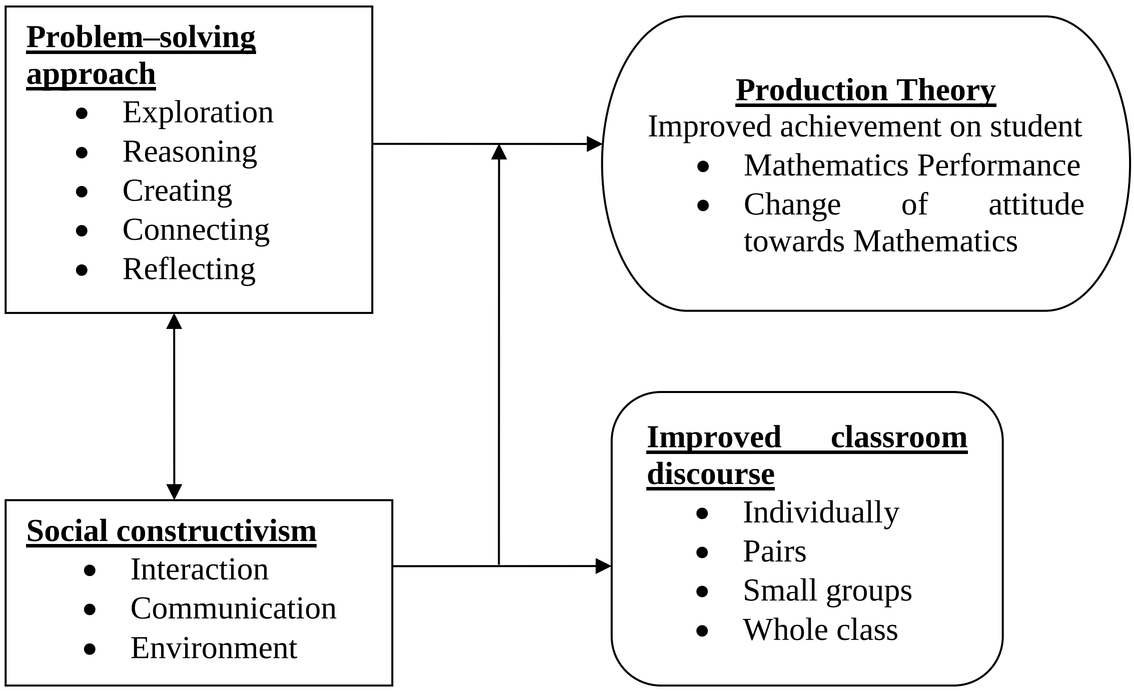


Figure 4: Theoretical Framework

Source: Adopted Social Constructivism Theoretical Framework

Figure 3 confirms the relationships of three theoretical frameworks intertwined in a high-quality learning environment where classroom discourse is paramount. The study adopted social constructivism theory in which students in the learning process are active participants concerned in learning through problem - solving. The teacher remains the most important resource person whose key work is reinforcement and directing learners to acquire new concepts through gradual enhancement. The continued procedure offers students all avenues needed for improving Mathematics achievement (Major, & Mangope, 2012). In this theory, teachers create an encouraging environment enabling students to obtain new information and custom facts gained to solve relevant and related problems. The active participation of students using problem–solving applying Polya's model led to improvement in Mathematics performance. Theoretical Framework considering social and cultural construction of learning indicates in context of human relationships and participation in Mathematics communities.

2.4 Conceptual Framework

A conceptual framework interconnected set of theories about how a particular phenomenon related to its parts. The study conceptual framework tried to be the basis to serve Mathematics understanding patterns that are correlational on interconnection across concepts, facts and information. There exist other learning experience components, which selectively accommodated in the classroom treatises. The Independent variable of this study was the problem–solving approach while the dependent variable was learners' Mathematics achievement score.

A conceptual framework based on building steps of difficult mathematical problems to simpler steps depending on students' observation of the problem. When the problem was broken learning activities task focused on students' attention on particular skill or ideas.

Teachers' role in classroom discourse involves organizing the environment for appropriate activities. These activities include selecting and structuring appropriate tasks to carry out in the classroom. The students actively participate in process of constructing knowledge. This makes Mathematics sensible to students in their developed ideas. In other words, students become active participants in the creation of mathematical knowledge rather than passive receivers of rules and procedures. These are classroom discourse for both teachers and students in problem – solving.

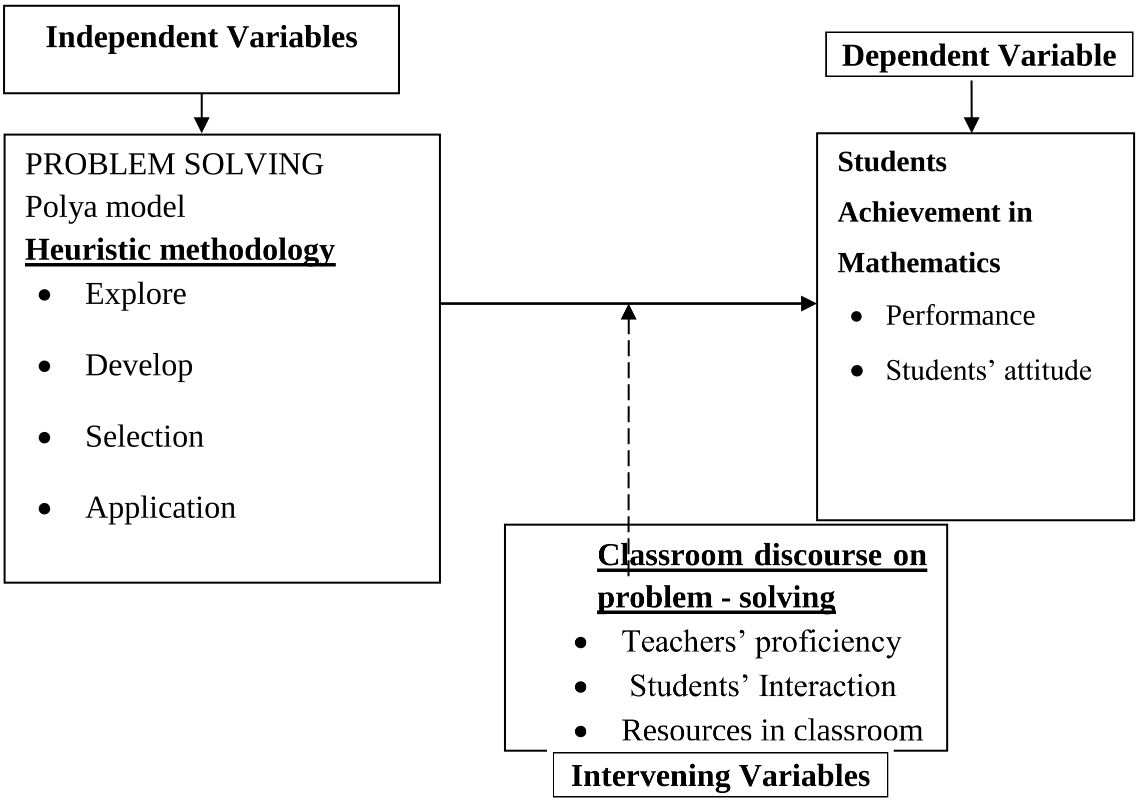


Figure 5: Conceptual Framework

Source: Researcher Adopted from Polya Conceptual Framework

The variables conceptually tied together as shown in Figure 4. The intervening variables included students' attitude change through the involvement of the teacher's competence. The constructivism theory recommends that knowledge individually acquire and construct socially as learners interpret present problems usually according to experiences. Therefore, problem – solving through involving learners to develop, explore, select and apply the mathematical concept. Student's understanding of the problem guided from beginning to end by the process of mathematical inquiry. This study adopted the investigation of teaching Mathematics through Problem–solving on the foundation of Polya's four stages, namely understanding problems, devising a plan, carrying out a plan and looking back. The conceptual framework was associated with the social constructivism theoretical framework that views learning and teaching as active meaningful inquiry, which built of knowledge by learner initiative in Mathematics classroom interaction and culture.

2.5 Research Gaps

The teacher class activities investigated using the Delphi questionnaire, which differentiated and state the conventional instructional strategy correlated with what happening in a problem–solving classrooms. The teacher's critical role was to the reflective phase of an inquiry-based lesson consideration on the process to generate results (Leiken & Rota, 2006). The teacher classroom behavior examined earlier found that frequency and duration of listening and watching from the beginning continue to summarise discussion without involving students in developing Mathematics. This had observed in Murang'a County where teachers have become learners rather than facilitators as shown by good performance in private secondary schools.

Research involving secondary school students seems to indicate that students will not sacrifice basic skills if taught using conventional methods. Research also makes clear that there is meaningless rote memorization of basic facts. This study established that basic skills are better developed when Mathematics is taught using problem – solving. Teaching Mathematics basic skills through problem – solving appeared to be both sound and viable in this study.

Researchers have shown that there was a lack of familiarity with word problem structures, which contributed to low performance in Mathematics. This study applying Polya's model helped students assisted to understand the problem well before suggesting what to do. The fourth stage of the model looking back helps both individual and group learners to examine their work and solutions of a group to promote understanding and retention. This study focused on students' efforts on problem-solving practices and learning strategies involving problem – solving in Mathematics.

The research done in this area of problem - solving seems not to have succeeded because of the nature of the students who were engaged in the research. Researchers have examined how to support student learning in reformed classrooms considering the application of ICT. These studies included other technology that does not form culture entrenched in the classroom community. These studies extensively referred to key explicit elements applying ICT to improve learning. This study redefines teacher's actions of communication, explanation and encouraging student engagement in mathematical tasks through the application of tools (Sullivan, Mousley & Zevenbergen, 2004). This study trained teachers involved to design and structure activities so that the learners interact to support their learning and building the Mathematics learning community.

Artzt and Armour–Thamas (2002) developed a model, which examined teachers' instructional practices where they distinguished between lesson phases and dimensions. Lesson phases included introduction, investigation and summary. This study encouraged learners to apply Polya's model of heuristic learning through student understanding of the problem, design the plan by preparation of learning environments, formation of social groups and intellectual climate in the classroom. They provided with minimum hints to carry out the plan individually and as a community of learners where interaction between teacher-student and student-student in communications are encouraged creating Mathematics culture.

Students' granted opportunities in Mathematics learning where they directly and decisively have an impact on the student's Mathematics achievement. Research in this area have been extensively done for several decades since 1945 where Brownell emphasizes instructional practices adopted to improve students' achievement in Mathematics through research done by Grouws and Cebulla in 2000 and updated in 2003. They came up with various instructional challenges including preparedness to learn, the meaning of mathematical terms, problem-solving, small group learning and use of calculators. This study suggests that students may learn better in both concepts and skills through problem – solving in Mathematics to improve students, attitude and performance. However, the main purpose of this study focuses on student development of skills and concepts before problem - solving in Mathematics used to extend practices and conceptual understanding.

Most teachers in Murang'a County complained about not completing the syllabus on time. This study's finding indicated that certain teaching strategies could carefully consider delivering Mathematics concepts for good achievement. These approaches have necessitated a drive of being worthy thought about as teachers strive to improve their teaching practices. The effects of teaching Mathematics through problem – solving on students' achievement given a thought. This study also considered whether incorporating students' interests and preferences could improve learners' attitudes toward Mathematics.

2.6 Summary of Literature Review

Teaching and Learning Mathematics through problem – solving on students' Mathematics achievement is an interesting venture worth studying. Students are expected that students will promote their confidence and ability to learn Mathematics through problem – solving. Students learn better in accordance to live applications of experiences in the real world. Mathematics cultured classroom community help in the communication of accomplished knowledge. The students bring their thinking into a classroom environment where they anticipated improvement working through a culture of interaction in Mathematics. Problem – solving built on teachers' expertise to set classroom conditions where students can move their informal Mathematics understanding to generalizations, formal representations of their mathematical thinking and recognition of others' capabilities.

The review provides effective Mathematics teaching and learning must in a challenging and engaging in a favourable environment for all students. Significant Mathematics learning would occur when sequential connections are effectively developed. Students' engagement in problem - solving in Mathematics enriches relevant problem understanding to improve on their achievements. The structured lessons to build on students' prior knowledge in the construction of problems offer entry points to gain better achievement, while given sufficient time and support to encultured in Mathematics.

The dream of the researcher was to share different ways of understanding Mathematics to attain good performance through problem–solving by working with learners in the Mathematics classroom in different ways for example grouping them, allowing them to speak and share their solutions to provide a different mathematical knowledge rather than traditional one. The learners should collectively experience problem – solving pertinently relevant to students' lives and reflect on expectations in the National Mathematics curriculum and Vision 2030 in Kenya. This was done through problem - solving by viewing and discussing the thinking and strategies in the solution through social interaction.

Experience of learning through problem - solving is by identifying what problem - solving looks, sounds and feels like by relating it to aspects of Polya's problem–solving process to experiences. The researcher disagreed that there were deep inter - connections between teaching impartiality and engagement in mathematical interaction. The cooperative approach in Mathematics teaching should encourage problem – solving. This study adopted the appropriate discourse of challenge to promote Mathematics equity and to combats phobia that Mathematics is hard but build students' confidence. Students' engagement in open social spaces through interaction in construction and discovery overcome Mathematics difficulty to make it enjoyable and liked compared to dull and supporting authoritative knowing. The vital development of critical consciousness gained reasonably by learning Mathematics. Therefore, learning problem–solving should be in 'I can do' where the environment generates mixed accomplishments in Mathematics, groupings help in the classroom community, interaction creates Mathematics culture needed, useful in life for long-lasting benefits on social equality in the society.

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Introduction

This section presented research methodology used in data collection and analysis. It consists of the research design, location of the study, target population, sampling and sample size of participants, research instruments applied in collecting data, pilot survey description, research validity and reliability, data collection and data analysis comprising with interpretation. Ethical considerations and logistical arrangements also described in this section.

3.1 Research Methodology

The purpose of this study was to examine the effects of using problem–solving strategies on students' Mathematics performance and attitude in secondary schools in Murang'a County. The presentation of quantitative methodology in quasi – experimental design supported improvement of Mathematics achievement. Questionnaires for teachers was a preferred technique designed using Delphi technique and students' attitude. Pre-tests and post-tests administered to collect achievement data. Experimental classes taught in their respective schools and provided with worksheets, which they use problem–solving strategy to work out solutions (Creswell, 2014). The study conducted in real classroom environment where students' behavior was normal in both control and experimental groups.

3.2 Research Design

The precise practical method used in this research study responded to research hypotheses or research questions referred to in the design (Privitera, 2014). The quasi – experimental method employed in which effects of independent variables used in class moderated by intervening variable to improve the dependent variables (Creswell, 2014). The researcher was interested in finding out whether teaching Mathematics through problem–solving strategies in classroom environment would manipulate students' attitudes, improve their achievements and promote collaboration (Trochim, 2006). The researcher used quasi – experimental design. The researcher preferred this design because it dealt with students in a school setup whose arrangement researcher had no control due to secondary schools' administrative factors.

The study specifically used Solomon Four Group design in quasi – experimental setting. Generally, participants' schools admitted students in Form one and classes formed (Creswell, 2014). Solomon Four Group design was preferred to overcome the problem of pretest sensitization while maintaining benefits associated with conducting pretest. In addition, avoiding other difficulties connected with posttest where pretests influence results considered. The design achieves this objective by randomly assigning participants' groups to either receive or not receive pretest, then randomly allocating these two to factors of treatment and pretest where four conditions created (Crano, Brewer and Lac, 2014). Therefore, two control and two experimental groups made to reduce effect of confounding variables. This accomplished by randomly allocating four schools per category to two experimental and two control groups.

Schools stratified depending on their four previous years' KCSE performances for placement in their respective categories. The design allowed the researcher to establish whether pretest itself had effect on participants before treatment commenced. The study determined that there is significant effect on students' achievement scores when problem – solving practiced by comparing scores of control groups and experimental groups after treatment. One of the treatment groups and one of the controls received pretest. The influence of pretest determined by contrasting difference in posttest scores between both groups that received educational treatment after intervention.

Two groups were administered pretest and later post-test.

E1 X E1

C1

E1: Experimental group one; X: Treatment & C1: Control group one

Solomon Four Group design enabled to combat many of the internal validity attrition of the research instrument as main reason for the applying design. Reasonable amount of control over variables exerted without having authority on pretest results. Solomon Four Group, which was 2 x 2 factorial design, applied where all groups received posttest. Pretests administered to one experimental and one-control groups. This provide major advantage of using Solomon Four-Group powerful design compared with others, since there was increasing generalization on findings (Cohen, Manion, & Morrison, 2011). The effect of treatment X reproduced in four different fashions compared to other experimental designs.

The design makes gives the researcher confidence about outcomes obtained during investigation. Solomon Four - Group design enabled the researcher to ensure that an in-depth study on variables underpinned in the study adequately evaluated. This means that it shows whether changes can occur in the studied variables. It identifies comparison group that remains as similar as possible to the treatment group in terms of pre-intervention characteristics. In essence, it controls any extraneous factors that are likely to affect the result. This is in agreement with Shuttleworth (2009), who ensured that effects of pre-test do not influence result of post-test in experimental design.

Further, one of the treatment groups and one of the controls received post – test only. Those that did not receive pretest commenced same time with those that received pre-test.

Two groups administered post-test.

E2 X E2

C2

E2: Experimental group two; X: Treatment & C2: Control group two

The experiment using Solomon Four Group design applied in such a way that experimental group E1 received pre-test and treatment X together with experimental group E2 and later post – test. The treatment commenced immediately after pre-test. The control group C1 received pre-test, whereas both C1 and C2 received post – test without treatment. All the four groups given attitude questionnaires before and after the period of treatment. The current study also addressed effectiveness of learning Mathematics through problem–solving method compared to conventional traditional strategies usually applied in secondary school level. The quantitative data on teaching strategies preferred by teachers were correspondingly collected using Delphi technique (Wright, 1999).

3.3 Location of the Study

The study conducted in public secondary schools in Murang'a County, one of the forty – seven counties in Kenya. Geographically, Murang'a County situated in former central province or Mount Kenya region. It has an area of approximately 2,558.8Km^2, with a population over 942,581 (Census, 2009). The County occupied by one of the largest communities in Kenya, the Kikuyu community. It also consists of other communities as minority. The county has serene environment conducive for learning in its eight sub - counties Murang'a East, Kahuro, Kangema, Mathioya, Kigumo, Kandara, Gatanga and Murang'a South. These sub- counties form eight main education zones. The general Mathematics outcome in secondary schools in these subs – counties during national examinations in Kenya are similar and the achievement continues to decline over the years. This challenge does not remain with students alone, but also Mathematics teachers feel not comfortable with the pedagogical skills they use in their classrooms.

The community feels challenged because their children are not able join science - oriented courses where Mathematics is the most essential subject. The evidence observed during annual prize giving days in the sub - counties where leaders keep on complaining that they have few students admitted at universities to pursue Medicine, Engineering and other Mathematics demanding courses. Murang'a county was picked through non - probability convenience sampling because of logistical reasons of access due to familiarity with researcher. Murang'a County stands as one of forty – seven counties in Kenya education administration zones in considering that community shared common attention in education of their students. Therefore, this County involved all stakeholders in education since high achievement in Mathematics stays essential for development of County human resource and economy. Mathematics learning made interesting to students through problem – solving strategies compared with conventional methods. The County has moderate performance comparable to the whole country. The schools' environment and learners were similar in all categories of schools in other parts of the country. Public secondary schools of Murang'a County evenly distributed in all the eight sub-counties between thirty- five and forty-five.

3.4 Target Population

A target population refers to total number of expected responses from which sample is drawn (Kombo & Tromp, 2006). The target population for the current study was all public secondary schools' students. Schools classified into four categories A, B, C and D whose population stratified according to performances prior to the study period. The total targeted number of secondary schools in Murang'a was 340 with total student enrolment population of 104,562. Those accessible form three students were 28,475. The target population also included 1365 teachers in the 340 secondary schools in the County. The public secondary schools classified as national, extra – county, county, private and sub-county did not participate as they are, but according to the previous years' performance.

Table 4 below illustrates data of the number of students and teachers in each type of secondary schools. This table does not demonstrate stratification since national Mathematics performance are independent of the school category.

Table 4: Murang'a County Secondary Schools Categorizations

Grades mean scores	No of Schools	Number of Students	Number of teachers
National	2	2816	19
Extra - county	27	17383	189
County	56	28815	336
Sub - county	232	54007	794
Private	23	1541	27
Totals	**340**	**104,562**	**1365**

Source: Murang'a County Education Office, (2018)

3.5 Sampling Procedures and Sample Size

A good representation of secondary schools in Murang'a County stratified with regard to previous four years' performance. Stratified samples obtained by separating population into smaller groups called strata. These were according to some given characteristics that was important to study (Bluman, 2014). Sampling frame was the total number of schools stratified with given groups in agreement to previous year's performances. Strata include A: $6 \leq X \leq 12$, B: $4 \leq X < 6$, C: $3 \leq X < 4$ and D: $1 \leq X < 3$ placed concurrently in their achievement mean score. Four secondary schools randomly selected from each given category. Four schools in each stratum randomly distributed into four groups. Cohen, Manion and Morrison (2005), maintained that studying overall classification of data would always be a matter of concern in making consensus. The sample results representing larger population considered in this present study appeal for good conclusion.

Form three classes were used study where two groups were experimental and two control groups. The four secondary schools were stratified randomly and assigned to either experimental or control groups. Where there was more than one stream, one class used without considering gender in mixed schools, because in most mixed secondary schools' classes were usually separated boys and girls. The reason for using form three class was that they understand basic concepts practiced and were familiar with the syllabus coverage for form one and two. The students had been together for two years and were free to interact with each other and they were able to form opinion on attitude towards Mathematics. Sampled schools indicated in the sample include only schools where teacher questionnaires administered in the purposes of comparing performance of schools, which were involved in the study of conventional methods used in learning and teaching Mathematics in County.

The independent variable was the problem-solving strategy through classroom activities and peer interaction. The independent variable through the treatment significantly affects the outcome of students' Mathematics achievement. The dependent variables included students' conceptual growth and attitude towards Mathematics. The teachers' preferred strategies considered in the manipulation of experimental using problem – solving strategy against traditional methods used in control groups. Applications of education instructional strategies classified into conventional methods. The experimental groups' treatment was exposition of problem-solving approach.

Table 5 shows the sampling procedure and the sample size used in the study in each category of secondary schools in Murang'a County.

Table 5: Murang'a County Grades Classification

Grades mean scores	No of Schools	Number of Students	School sampled	Number of students	Number of teachers	Teachers sampled
D:Less than 3	225	13329	4	116	832	4
C:3 < X < 4	60	5923	4	149	258	4
B:4 ≤ X < 6	35	4381	4	123	172	4
A:6 ≤ X ≤ 12	20	4842	4	156	103	4
Totals	**340**	**28475**	**16**	**544**	**1365**	**16**

Source: Murang'a County Education Office, (2018)

Table 5 illustrates the sample sizes for strata A, B, C and D, each with four secondary schools. The students sampled includes those who responded to the study. Students who participated in the pre-test were 268, while those who participated in posttest were 544 students. The numbers varied because of students who consented to take part in study and schools' enrolment were not evenly distributed particularly in National and sub –

county schools. The teachers of sampled classes per school conveniently worked as a team in collecting data. Teachers who were dealing with the study interventions were individually prepared and discussion approaches harmonized within the research period for consistency. The four schools from each given stratum form one group to avoid biases of entry points of the students from primary schools.

3.6 Research Instruments

Four instruments used to operationalize the data collection. Pre-test and post-test measured students' Mathematics performance before and after treatment. These instruments developed by the researcher in consultation with Form three teachers and examiners with KNEC. Bloom's (1956) taxonomy classifies levels of cognitive domains as comprising pretest and posttest instruments, which developed covering four topics with five questions each. The experimental class used problem- solving while the control one adopted traditional instructional strategies. The Bloom's taxonomy cognitive hierarchy levels used to establish learners' level of entry. The development of deeper understanding of concepts through procedural skills developed after intervention assessed through post-test. The teaching and learning strategies were more than twelve but the researcher considered problem-solving strategy as superior to all the others. Teachers preferred some strategies to others and their opinions collected using Delphi techniques, which done twice throughout the period of study.

3.6.1 Research Instruments for Students' Attitude

Questionnaires were used to collect data on the attitude towards Mathematics used in this study was adapted from Myanmar version consisting of twenty items, which deal with Mathematics Behavior (MB), Mathematics Confidence (MC) and Mathematics Engagement (ME). A five-point Likert scale type scale questionnaire including totally disagree, partially disagree, uncertain, partially agree and totally agree (Appendix F). The participants completed the questionnaire before and after the treatment.

3.6.2 Research Instrument for Pre-test

A pre-test considered to be any questionnaire or test that participants were required to complete prior to implementation of treatment (Slavin, 2007). The researcher identified some basic concepts within the current teaching materials and trained select teachers as national examiners to set questions. The guidelines given by the syllabus developer, Kenya Institute of Curriculum Development (KICD) used syllabus developed resources suggested (MoE, 2002). These questions moderated by experts, senior examiners and teachers from KNEC who evaluated the relevance of these topics. The pre-tests helped the researcher to collect data on participants' level of performance before intervention at each level in each school from the Experimental and Control groups. The pre-tests comprised 20 short items on various topics including numbers, algebra, number patterns, geometry and measurements (Appendix D). Students participated during pre-tests were from experimental group, E1 and control group, C1. This followed research design of study employed in accordance with Solomon four-group design to determine the effects of the intervention.

3.6.3 Research Instrument for Post-test

A model for instructing students was developed. This resembled that of Artzt and Amour-Thomas, (2002) in which lesson phases and lesson dimensions involved students. The introduction established students' readiness in learning. This done through students' engagement and investigated their readiness to learn new concepts. The treatment involved students in constructing new meaning through exploration, explanation and summaries of their work. The learners assisted each other to integrate what they have learned and extended through classroom interactions. Similarly, post-test questions set by experienced teachers, moderated by experts whom include senior examiners of KNEC (Appendix E). Pre-test and post-test have bene mulled over to compare changes of different groups' aspect on Mathematics concepts gained during intervention of the study. The research design employed advocated that all groups received post-test instantly after treatment. This was to determine whether experimental action has any noticeable impact on the participants' performance.

3.6.4 Research Instrument for Teachers

The teachers' questionnaires provided own view information about the subject of study by respondent on various issues (Johnson & Christensen, 2012). Teachers' questionnaires developed to evaluate the objective of the study that using problem-solving strategy produces better Mathematics' achievements for students in secondary schools in Murang'a County compared to other preferred conventional strategies. The questionnaire consists of three vital sections (Appendix G). The first section tries to find out more information on characteristics of teachers for instance teaching qualifications, Mathematics performance in their school and teaching experience. The second part of teachers' instrument constructed using Delphi, which two questions about each strategy in the instrument test each method. The instrument engaged the participants on issues of methods used for teaching in confidentiality and disclosed relevant information for conventional methods.

The research questionnaires divided statements according to strategies for tranquility analysis of participants' responses. These statements were constructed in twelve crucial strategies of teaching Mathematics which include cooperative, demonstration, discussion, exposition, game, guided discovery, homework, ICT including audio, laboratory, problem-solving, question/ answer and supervised practice. The statements accompanied by five-point Likert scale. Table 6, shows teaching strategies used by teachers' questionnaire statements distribution per strategy.

Table 6: Teaching Strategies Used by Teachers' Questionnaire Statements Distribution

	Strategy	Statements		Strategy	Statements
1.	Cooperative	16, 22	7.	Homework	2, 14
2.	Demonstration	7, 12	8.	ICT	13, 15
3.	Discussion	4, 5	9.	Laboratory	9, 17
4.	Exposition	1, 20	10.	Problem–solving	16, 21
5.	Game	19, 23	11.	Question/answer	3, 6
6.	Guided discovery	8, 11	12.	Supervised practice	10, 18

Source: Field survey questionnaire, (2018)

Table 6 shows there were two questions for each strategy for confirmation of what the teacher responded respectively to each strategy. The purpose of these teachers' questionnaires was to enable classification of their views and opinions. The two questions constructed to specify how each strategy has been employed with statements, forming the answers 'is' and 'should be' responses. The statements on particular strategy were not following each other consistently in order to enable confirmation done in strategy rating through walk unsteadily.

3.7 Piloting of Research Instruments

Pilot study is a miniature scale justification of main research preparation carried out to investigate results (Polit, Beck & Hungler, 2001). The fundamental nature of pilot testing gauged power of research instruments in terms of content coverage and validity construction. The pilot study conducted to ascertain that instruments captured necessary information effectively. This study used quantitative method that includes descriptive and inferential statistics. Piloting of instruments carried out in public secondary schools not involved in the actual study in Murang'a County, Kenya. These schools purposely excluded from the study-sampling frame to ensure clarity wording of items, statements and instructions used in questionnaires.

The questionnaire for teachers adopted on the improved Master of Education degree in Nyeri County (Gakinya, 1987) producing Spearman's Rank correlation coefficient of 0.8. This developed further to make instruments more reliable and valid during data collection using cognitive practices. Basic Mathematics tests already developed used to test the validity of the results. The teachers using statements in the questionnaire were anonymously able to confront each other with their knowledge and views on strategies they use in teaching Mathematics.

3.8 Validity of the Research Instrument

The researchers indicate that validity can "draw meaningful and justifiable inferences from scores about sample in which population inferences are drawn" (Creswell, 2014). The internal and external validity concerns are threats in experimental research since they are likely to exist but controlled (Neuman, 2003). The internal validity threats due to experimental procedures applied on treatments and experiences of the participants were able to achieve possible goals for this research model. The controlled harmonized pre-test and post-test strategies through the help of teachers teaching in secondary schools and contribution of KNEC examiners were of benefit. Purifying experimental processes and achievement of scientific analysis required skillful control of most irrelevant variables through discussion with teachers involved directly with participants.

The treatment took ten weeks, which was a short period to affect history, maturation and mortality of participants. The period has minimum change of participants' cognitive development. The experimental and control groups homogeneity was maintained by short time bound. The researcher achieved qualitative validity by employing learners of the same age in order to authenticate accuracy of findings. Instruments for measuring pre- test and post- test developed by teachers handling learners in Form three and approved by national examiners (Creswell, 2014). The qualitative reliability points out that research approach was consistent across different researchers in different projects' considerations. The validity of assessment was a function of practices made of the information (Romagnano, 2006). The content information used to determine how much students know in Mathematics. Therefore, content validity in this study was determined by confirming the content covered in the instrument which are topics covered in Kenya secondary syllabus in form one and form two observed by secondary schools' teachers.

3.8.1 Reliability of the Variable

Yin (2009) recommended that qualitative researchers need to manuscript the events of their studies and as many steps of procedures as possible including setting up a detailed study protocol and database**.** Reliability measured with consistency and dependability using instruments over time. The correlation coefficient between measuring instruments, often termed as reliability coefficient, requires determining degree of association (Ellen, 2011).

Pre-test and post-test instruments gauging students' Mathematics achievements on test items developed and moderated by experienced teachers. The reliability development planned to use interview protocol, questionnaires and observations for asking questions and recording answers. These instruments discussed by supervisors and teachers who comprised in research assistants before qualitative and quantitative data collections. When the assessment discussed with teachers and supervisors, it produced accurate and consistent evidence in the process of pretest, intervention and posttest. The researcher at a Spearman's correlation coefficient of 0.8 applied Delphi technique measuring students' performance. This further modified to reestablish validity and reliability during current situation data analysis (Gakinya, 1987).

3.9 Data Collection Procedures

The research conducted using quasi-experimental design in sixteen secondary schools in Murang'a County in which eight were control and eight experimental groups. The topics purposely selected from secondary Mathematics syllabus since most people consider the most difficult topics from students' survey on the subject. Solomon four-group design controls most sources of invalidity. The design was involved when randomizing assignment of experimental and control groups. Table 7 shows data collection matrix of four groups. The results in the Four Group design were as has been suggested by Solomon that two groups were pretested whereas the other two groups were not pretested before commencement of experiment. The pretests measured checked whether two groups were equal at beginning of experiment. The two groups assumed similar to the pretested groups and treatment of experimental groups commenced at the same time.

Table 7: The Solomon Four Group Design Matrix

Category	Data	Intervention	Post-test
E1	Pretest	X:treatment	posttest
C1	Pretest	₦	posttest
E2	No pretest	X:treatment	posttest
C2	No pretest	₦	posttest

Source: Researcher, (2018)

E1 and C1 presents experimental and control groups receiving pre –test respectively.

X: stands for treatment, ~~N~~: No treatment

E2, C2 experimental, and control groups received post – test only.

The study engaged two variables including independent and dependent variables. The independent variable was problem-solving strategies during the intervention used in classroom discourse subjecting learners to social constructivism. The dependent variables in the study were students' cognitive and conceptual development; students' attitude towards Mathematics; and teachers' perception of the methods preferred in learning and teaching Mathematics in a conducive environment. Pre-test administered to one experimental and one-control groups composed of four different schools according to their category. The pre–test weighed base line to decide whether there any significant differences in the mean scores between the control group and the experimental group that received intervention. The marking done by same researcher in all groups' consistency of grading and marks awarding.

Post-test items administered to all four groups comprising of sixteen groups on the last day of intervention week. This research took the whole class as an experimental unit in each school and the existing stratum in two randomly assigned levels as either experimental group or control group. The teachers' Delphi questionnaires and students' attitude were made following Likert-type scale format score of 1 totally disagree, 2 partially disagree, 3 uncertain, 4 partially agree and 5 totally agree. Study attitude questionnaire was composed of three different areas of Mathematics including Mathematics behavior, confidence and engagement.

3.10 Data Analysis Techniques and Procedures

The data analysis addressed each hypothesis (Creswell, 2014). Statistical data collected presented in tables and analyzed using inferential and descriptive statistics. Descriptive statistics employed included frequency distribution tables and graphs, percentages, mean and standard deviation. Independent Spearman's Rank Correlation coefficient, t-tests, z- score and Analysis of Variance (ANOVA) used in comparison of the data. The data accumulated were analyzed using excel or SSPS version 23 to test stated hypothesis ANOVA inferential statistics. The ANOVA used to test hypotheses of performance in post-test achievement test where more than two groups are involved. The reason for using the F – test is that the four means obtained during post-test can be compared simultaneously. The analyzed students' responses for both descriptive and inferential data at significance level set at $\alpha < 0.05$. The effect sizes value interpreted using Cohen's (1988) categorization of effect sizes. Cohen's d calculated as difference between the means divided by the pooled standard deviation.

Cohen's d $= \frac{meandifference}{s\tan darddeviation} = \frac{M_2 - M_1}{\sigma}$,

M_1: first mean; M_2: second mean and σ: standard deviation

This usually used as a multivariate analysis where t-test and ANOVA used to confirm results applying Cohen's d effect test (Ruscio & Mullen, 2012). The general guidelines are small 0.2, medium 0.5 and large 0.8 when interpreting effect before and after an intervention.

In conclusion, two groups each composed of four strata elements received pretest before intervention commenced analyzed. Two groups exposed to treatment and tested in order to compare the effects of intervention as well as two control groups. This controlled general history of events that may have contributed to effects of problem – solving. The statistical inferential tests applied to produce authentically viable results interpretation.

3.11 Ethical Considerations

Ethical issues may stem from kinds of problems investigated from students and teachers in respective secondary schools of Murang’a County. Methods used to obtain valid and reliable data and the concerns of respondents addressed through tests moderation. The study conducted where a strict code of conduct upheld. This includes assurance to respondents’ confidentiality and their cooperation as important. The respondents’ real names protected by using admission numbers only for students and codes for teachers. The participants’ identification names not used during while reporting findings. Consents sought from all respondents in this study and consent forms signed before starting data collection. Before commencement of data collection, arrangements and preparations made as stipulated by the Mount Kenya University research policy and Ministry of Education. The Kenya Administration permissions to collect data from schools in the County also sought. Plagiarism avoided through acknowledging of the work done by other researchers on the topic under investigation.

Introduction letters given by National Commission for Science, Technology and Innovation (NACOSTI) to carry out the study. This is a compulsory authorization granting researchers admittance to any area in Kenya for data collection. Permission from parents through principals to conduct an experiment on their children obtained before commencing research. This supported involvement of the main respondents who were students for treatment. Obtaining consent and cooperation of Mathematics teachers whose classes sampled was important to establish required statistics. The teachers’ assistance in investigations and using their classroom facilities was consented.

The quality of consent from students and teachers also included groundwork for good rapport with respondents.

In carrying out research, respondents assured of privacy of information that they provided without coercion and in varied opinions. The participants voluntarily signed consent forms before participating in the study. The participants received clear explanation before the investigation started. They give first-hand information of the benefits, rights, risks and dangers before they were involved. The consequences of their participation stated while signing informed consent. Through request of participants, they guaranteed to receive a copy of the final report. Supported by Cohen, Manion and Morrison (2005) that informed consent protects and respects rights of self-determination as researcher places some responsibility on the participants.

The respondents supplied information without their identities on research instruments for both teachers and students. The researcher used admission numbers for students and identification codes for teachers for categories. The participants guaranteed that data collected treated with utmost confidentiality in storage. The data analyzed using combined categories in each of the four stratified groups. This aligned with signed informed consent where the participants were voluntarily involved, and they had a right to refuse or withdraw.

According to Kombo and Tromp (2006), the specialist will uphold most extreme respectability attributes as indicated by the traditions of society and fitting set principles in the field of research. Therefore, charming look and satisfactory quirks all through the investigation procedures examined. The protocols strictly observed and adhered to in order to avoid interference of normal schools' schedules.

The students in other streams not informed to avoid the competitive nature of respondents so that the research conducted in a natural environment and the teachers continue in normal teaching. This was because most of the schools selected had more than one stream, so the class sampled was to work like others. The students in the schools in the same stratum not informed since some schools were in different sub - counties within Murang'a County. Therefore, no information about the respondents revealed in written or any other form of communication. This greatly assisted the researcher to obtain unbiased responses from participants.

The raw data collected were stored for future reference. After the data was analyzed using computer, the printouts filed as hard copies stored in flash diskettes as well as VDs in soft copies. The study explored educational management in practice as a condition of its implementation. It hoped that the final report would be disseminated to benefit the schools that took part as well as all secondary schools in Murang'a County.

CHAPTER FOUR

RESEARCH FINDINGS AND DISCUSSIONS

4.0 Introduction

The study focused on effects of teaching through problem-solving strategy on students' Mathematics achievement in secondary schools of Murang'a County. Findings of this study operationalized by students Mathematics achievement at secondary schools, which were students' performance and attitude. The teachers' preferred strategies considered to evaluate teachers' opinions on the methods of teaching Mathematics usually practiced. These divided into three indicators as different entities in analysis. The number of students who responded to the pre-test was 275 while the number that responded to the post – test questionnaire was 544 since some students did not take part during pre-test. Therefore, all the participants' responses used in data analysis because the number was not too large to affect the variance of data. The research was quasi – experimental where each response counts during data analysis.

The study conducted to answer the question of whether teaching through problem–solving strategy improves the students Mathematics achievement. Is there a change in students' confidence in Mathematics through classroom interaction? What is the influence of giving Mathematics instructions using problem–solving approach on students? These answers sought to relate Mathematics achievement comprising of attitude and performance in public secondary schools in Murang'a County. Prototype lesson using problem – solving developed to assist teachers to improve Mathematics achievement.

This study conducted to achieve the following objectives. First, to establish the preferred conventional strategies in teaching Mathematics over problem–solving strategies in teaching. Second, to compare students' Mathematics performance for those taught using problem–solving strategies with those taught using conventional strategies in secondary school. Third, to assess the students change in attitude towards Mathematics when taught through problem–solving strategies. Fourth, to develop prototype lesson plan for teaching Mathematics using problem–solving in public secondary schools in Murang'a County. These objectives form the foundation of this chapter, as they were expounded using collected data analysis.

Further, the research hypotheses formulated and discussed depending on data results from secondary schools in Murang'a County. The first research hypothesis was stated as Mathematics knowledge gained through problem–solving approach has no statistical effect on student' performance in Mathematics in secondary schools in Murang'a County. The second research hypothesis stated that there was no significant difference in students' attitude towards Mathematics while there were students taught using problem–solving approach and those taught by means of conventional strategies. A number of tables were included and discussed in this chapter representing the outcome of this research to relate the hypotheses to data collected.

4.1 Demographic Information

The selection of secondary schools was according to their previous four years' performances in Kenya National examinations. Although entry point of students from primary schools to different secondary schools done on merit of their primary school examination, it was not necessary to employ this, because their Mathematics performance does not tally with secondary schools' students admitted at entry point. The admission of students to schools in category A were usually in National and Extra – County, category B were county schools and those in category C and D were mostly sub–county and private schools. The procedure of enrolment through central placement of students from all over the country in either national or extra - county goes beyond this study. There were two sets of questionnaires; pre –test and post – test; with 20 items each. These questionnaires administered to two groups at different levels in their schools.

Pre-test achievement done by both clusters in experimental and control before intervention took place in the respective schools. One experimental cluster of schools, E1 and one cluster of schools in control, C1 received pretest prior to commencement of intervention. Experimental, E2 and control, C2 groups did not join in pre –test attainment. The intervention began after pretest to both experimental groups E1 and E2, using problem – solving whereas both control C1 and C2 continued to be taught normally. All groups did post – test immediately after intervention. Table 8 illustrates participants of both control and experimental clusters in their respective category of public secondary schools sampled. This means that the four schools in each category and the number of students were those who consented for the study.

Table 8: Distribution of the Students Respondents by Research Group

Category	Pre-test		Post – test only		Total
	Experimental E1	Control, C1	Experimental E2	Control, C2	
A	33	45	33	45	156
B	35	35	27	26	123
C	40	40	32	37	149
D	20	27	34	35	116
Total	**128**	**147**	**126**	**143**	**544**

Source: Murang’a County Field data, (2018)

Table 8 shows the total number of students from each category as well as each group who participated in the study. The numbers of experimental students were 254 and that of control students 290. The variation existed because it was the exact number of those consented students who count in this study as per the secondary schools sampled. The experimental schools requested to involve only a limited number of students in the study whose participants randomized using class list as sampling flame for uniformity of marking. This done because some classes had more than sixty students particularly in Category A and B. All students were involved, but for marking, only the participants engaged by the researcher in assisting teachers helped in marking other students. The study considered sixteen Mathematics teachers from sixteen secondary schools used in the study. Eight schools in categories A, B, C, and D used for experimental demonstration through a guided intervention. The other eight schools in categories A, B, C and D used as control groups and they were not involved in the training to avoid external validity.

4.1.1 Demographic Data of Teachers Respondents

The distribution of sampled Mathematics teachers was according to the four natures of schools, their qualifications and experience. The mean scores in their schools obtained in 2017 also considered. The sampled schools' information was necessary since social policy issues were important in this study. The findings based on social constructivism and used to improve Mathematics learning situations using problem –solving strategy. Approaches in classroom activities depend on teacher's performance and efficiency, who had obtained required qualification and experience through problem -solving. Usually, the logical knowledge role and importance of ethics in teacher evaluation in confidentiality was reserved (Cohen et.al, 2011). Only one stream per school sampled presented in table 9.

Table 9: Distribution of Teachers Respondents per School According to Their Experience and Qualification

Category	Mean score	No of teachers			Experience < 3 years	Experience> 3 years
		Diploma	Bed	masters		
Sub-County		1	4		1	4
County			6	1	2	5
Extra-County			2			2
National			2		1	1
Total		**1**	**14**	**1**	**4**	**12**

Source: Murang'a County Field data, 2018

Table 9 presents teachers' distribution in each category of schools. The numbers of teachers in sub – county and county schools were more than the national schools and extra – county schools because the lower categories have more of these schools. There are two national schools, eighteen extra – county, one special, thirty- two private, eighteen county schools and two hundred and sixty - nine sub - county schools. The analysis of data shows that 6.25% of the teachers are diploma holders, 87.5% trained graduates and 6.25% were masters-holder teachers, which gives total 100 % of teachers with pedagogical skills. These data revealed that most teachers are also experienced with more than three years. These data demonstrated that using good approach in teaching and learning Mathematics, students should produce better grades in their achievement.

4.2. Presentation of Research Findings

This section presents research findings in agreement with objectives of the study stated as conventional methods being preferred by teachers in learning Mathematics; problem–

solving on students' Mathematics performance; problem–solving changes students' attitude towards Mathematics and develop a prototype lesson plan.

4.2.1 Conventional Strategies Preferred in Teaching Mathematics

The first objective of the study was to find out the preferred conventional strategies over problem–solving strategies used in teaching secondary school Mathematics in Murang'a County. In other - words to compare possible teaching methods in Mathematics according to teachers in Murang'a County.

> *This considered the hypothesis that there is no significant positive correlation (at α = 0.05) between the conventional strategies used and the preferred strategies by Mathematics teachers in secondary schools in Murang'a County.*

Teachers overall group respondents compared as follows: the mean scores 'is' and 'should be' for each strategy was calculated. They were also ranked to show the 'is' and 'should be' to determine most preferred strategies although respondents were aware of the most appropriate pedagogical skills from whole group. Through informal interviews, the reasons given although they were not part of this research. The rankings of strategies and means of 'is' and 'should be' ratings are tabulated for the 16 teachers involved in the study. The mean rankings of the conventional strategies used by teachers during pre-test indicated in Table 10.

Table 10: Mean Rankings of the Conventional Strategies Used by the Teacher during Pre-test

Strategy	'is' Mean	'Should be' Mean	'is' Rank	'should be' Rank
Cooperative	3.4	3.93	3	4
Demonstration	3.54	3.97	1	2
Discussion	3.17	4.06	6	1
Exposition	3.12	3.44	9	10
Game	3.15	3.85	7	5
Guided discovery	3.14	3.70	8	8
Homework	2.90	3.40	11	12
ICT including audio	2.46	3.42	12	11
Laboratory	3.04	3.76	10	6
Problem – solving	3.30	3.73	5	7
Question / answer	3.33	3.63	4	9
Supervised practice	3.41	3.96	2	3

Source: Murang'a County Field data (2018)

Note: *Kendall rank – order correlation coefficient between 'is' and 'should be' is 0.645 and Spearman's rank of 0.72 which are close.*

Table 10 illustrates that demonstration 'is' most used strategy (M = 3.54) and discussion (M = 4.06) is most preferred strategy whereas ICT (M = 2.46) and homework (M = 3.40) are rarely used and preferred, respectively. The implication of teaching Mathematics through demonstration requires the teacher to calisthenics model sums on the board, and then students merely follow the patterns. The model or pattern promoted by teachers or textbooks strictly adopted and copied by the learner hinders concept development. According to Bruner, constructivist theoretical framework suggests that learning is an active process where learners construct new ideas and concepts pursuing course of current and past knowledge (Bruner, 1996). Demonstration method supports neither child nor Mathematics subject learning since it allowed students' mind stuffed with information without understanding. Although the preferred method does not find place in Mathematics subject teaching for better performance. The study later developed a model lesson plan to help teachers use problem–solving strategy to improve learner thinking and creativity.

The teachers conceded that they talk too much but they should allow the learners to be involved in the learning environment. Most teachers contended that problem–solving method "should be" used very frequently although they rarely use it. The ranking of discussion "is" position six (M = 3.17) and "should be" ranked one (M = 4.06) confirms that class interaction is an important component of good learning environment. The learning activities arranged in a way that both the teacher and the student is engaged in dialog known as Socratic learning (Herbert & Wearne, 2003). Mathematics learning in this study focused on development of the framework for reasoning processes as described by social constructivists' models.

The study was investigating students' achievement and understanding of Mathematics by finding out whether problem – solving significantly improve performance. Teachers made aware that students' achievement contributed by how they are involved in constructing knowledge themselves. According to teachers, there was confusion between problem – solving (M = 3.73) and discussion (M = 4.06) that is why it was ranked first as a preferred method. The Spearman's rank correlation coefficient was 0.72, which shows strong correlation between 'is' and 'should be' which predict that there is teachers' attitude that they are doing the right thing when they engage learners.

There is clear evidence that teachers' attitude must change to improve learner's achievement through integration of using problem–solving techniques. The rankings of “is” and preferred “should be” responses for the combined groups also ranked in order correlation coefficient (tau = 0. 645) with z = 2.93 and p = 0.0034 which is significant at 0.05 level. The spearmen rank correlation coefficient of 0.72 point out that null hypothesis should be rejected and accept alternative hypothesis.

> *Null hypothesis rejected since 0.72 is more than critical value of 0.591. Since there is enough evidence providing the correlation between the rankings of ‘is’ and ‘should be’*

It was interesting to note that discussion, demonstration and supervised practice methods ranked the top three because they are similar, as they require students’ participation in following certain process without much creativity and resourcefulness. Teachers argued that due to low income in public secondary schools, there was scarcity of ICT and audio-visual equipment. Teachers added that workload was overwhelming to give time to creativity, which produce handmade models for demonstration. Generally, there was lack of sincerity and diligence among teachers who wish to complete the syllabus or syllabi earliest without putting genuine efforts. The teacher might use variety of instructional methods to enhance effective pedagogies on developing higher order thinking and retention to improve on Mathematics achievement of student in secondary schools.

The high mean score of cooperative (M = 3.4 and M = 3.93), game (M = 3.15 and M = 3.85) and problem – solving (M = 3.30 and M = 3.73) reveals that teachers were on suggestion that students “should be” given opportunities to interact and encouraged to find their own solutions. Students offered chances to share and compare their methods and answers improve classroom discourse. The overall preference changed teachers’ perspective particularly those who were involved in intervention.

Accordingly, central principle on instruction where reforming pedagogy in Mathematics have been that students benefit from correlating thinking deeply on multiple solutions methods used during classroom discussion. This study aligned to 21st century educational reforms of Mathematics education that the teacher acts more as facilitator who guides student without giving a lot of vital constituent support. Students provided with enough opportunity to share and compare their own thinking as reflection on problem – solving continues. Students enjoyed Mathematics in classroom discourse promoting collaborations creating mathematician communities.

Table 11 shows the mean scores and rankings ‘is’ and ‘should be’ for each strategy computed after intervention.

Table 11: Rankings of the Conventional Strategies and Their Means after Intervention

Strategy	‘is’ Mean	‘Should be’ Mean	‘is’ Rank	‘should be’ Rank
Cooperative	3.6	4.85	3	2
Demonstration	3.0	4.20	7	10
Discussion	2.6	4.10	11	11
Exposition	3.75	4.70	2	3
Game	2.72	4.25	9	9
Guided discovery	2.90	4.4	8	8
Homework	2.60	4.5	10	6
ICT including audio	1.15	3.7	12	12
Laboratory	3.44	4.6	6	5
Problem – solving	3.45	4.90	4	1
Question / answer	4.0	4.65	1	4
Supervised practice	3.41	4.45	5	7

Source: Murang’a County Field primary data (2018)

Table 11 demonstrates that rankings of strategies and their means worked out after intervention. Cooperative method changed rankings (M = 3.6 and M = 4.85), question answer (M = 4.0 and M = 4.65) and problem – solving (M = 3.45 and M = 4.90) were better methods according to Mathematics teachers which altered because of those involved during intervention. This also modified by teachers' change of opinion after intervention aligned to Delphi techniques. They were aware of students' behavior in their classrooms after they reminded about strategies of teaching during intervention. Exposition (M = 3.75 and M = 4.70) was also rank position 2 and 3 respectively instead of demonstration and discussions as it was before pre-test. The Kendall Tau 0.66 and Spearman correlation 0.83 had shown that z - value of 2.987 with p = 0.0028.

> *Null hypothesis was rejected that there was positive correlation (at α = 0.05) between the conventional strategies used and the preferred strategies by teachers of Mathematics in public secondary schools in Murang'a County.*

There was enough evidence that there was a positive correlation between statements rankings of 'is' and 'should be'. This has made known that problem–solving approach contributed positively to preferred methods. This confirmed that conventional methods related to problem – solving have added significance in Mathematics teaching.

4.2.2 Students Performance in Mathematics Taught through Problem–solving

The second objective compared students' Mathematics performance of those taught using problem–solving strategies in contrast to those taught using conventional method in secondary schools in Murang'a County.

> *Second hypothesis examined whether there was statistically significant difference between means scores of students in control group who taught using conventional methods and those taught through problem–solving strategy in experimental group during pre-test and post – test achievements scores.*

There was change in rating as well as ranking after intervention. The conceptual and cognitive growths in Mathematics determined by achievement test complying with pre-test and post-test questionnaires. Students expected to learn Mathematics through problem–solving strategy while others through conventional strategies. This study suggested that students' conceptual understanding developed better using problem–solving rather than procedural knowledge using conventional methods in secondary schools in Murang'a County.

Solomon four group designs reveals that various combinations of tested and untested groups with treatment and control groups' results weighed against each other. These allowed the researcher to deal with extraneous factors that may have or have not influenced present results. E1, C1, E2, and C2 were the same in all four categories according to their previous national examination. They draw from four similar schools with the same standards. These were done by stating sub-hypothesis regarding performance of assorted permutations.

> *The first sub-hypothesis that there was no significant difference on Mathematics achievement (performance) with pre-test mean scores between experimental group E1 and control group C1.*

In everyday life students learn Mathematics, this study examined Form three students who had already learnt Mathematics in Form one and two of secondary school. Pretest and posttest items included these levels. Most teachers continued longing for better Mathematics achievement for their students. There was evidence of weak Mathematics performance basing on previous years KNEC examinations results. The pre-test measured whether students were at same level in their categories and groups. The effects determined in the following tables affirmed the second objective and hypothesis on students Mathematics performance.

The pre-test involved experimental E1 and control C1 groups. The results of pre-test performance per stratum shown in table 12.

Table 12: Pre-test Performance per Category Experimental and Control

Experimental				Control		
Category	No of respondents	Mean	Standard deviation	No of respondent	Mean	Standard deviation
A	33	54.67	6.79	45	50.44	8.79
B	35	44.94	11.09	35	42.57	10.74
C	40	28.75	13.36	40	30.48	7.71
D	20	25.55	15.15	27	14.70	9.65
Combined	**128**	**39.36**	**16.39**	**147**	**36.57**	**15.79**

Source: Murang'a County Field data (2018)

Table 12 makes known that there was no difference in mean scores between group except from categories, which were due to entry point at secondary school from primary schools for instance comparing categories A and D. The categories A, B, C and D pre-test had Cohen's d 0.54, - 0.22, - 0.16 and 0.80 effect showing that were moderate, near zero and moderate shows no significant mean difference during the pre-test. The objective determined substantial role-played by problem-solving approach on student achievement scores rather than conventional methods at secondary schools in Murang'a County before treatment.

Study employed Solomon Four - Group design, which enabled two clusters in each category to be examined on pre-tests as recommended by Borg and Gall (2006). Experimental group (E1) and Control group (C1) pre-test conducted before commencement of the treatment to determine entry point of learners. These clusters participating during pre-test tested in their particular schools. After administration of prior assessment in schools involved, groups in E1 and C1, preceding experiment on clusters began. The mean scores and standard deviation of two groups E1 and C1 pre-test data computed show that learner were at same level.

Outcomes on students' performance mean scores of combined experimental E1 and control C1 groups' pre-tests documented in table 13.

Table 13: Students Pre-test Performance per Group

Combined group	NUMBER	MEAN	VAR	STDEV	Standard error	95% Confidence	T–value
E1	128	39.36	268.66	16.39	1.454	36.52 - 42.20	0.174
C1	147	36.57	249.22	15.79	1.307	43.02 - 39.12	
Combined	**275**	**37.87**	**259.26**	**16.01**	**0.969**	**35.97 - 39.77**	
Difference		2.79					

Source: Murang'a County Field study (2018)

VAR: Variance

STDEV: Standard Deviation

Table 13 shows that students from experimental group E1 performed slightly better than control groups C1 students in pre-test Mathematics performance, although insignificant. This affected by the mean score of schools in category D schools, which performed lowly, since usually mean is affected by extreme values.

> *Hypothesis tested that there was no significant difference in mean scores between achievement of control C1, and experimental group E1, in pretest.*

Table 13 reveals that independent statistical t-test had insignificant difference in mean scores of combined pre-tests of experimental (E1) and control groups (C1) formed t (274) = 0.174, $p = 0.1528$ at $\alpha = 0.05$ where $p < 0.05$. This clearly indicated that students' Mathematics performance during pre-test were comparable. The level of understanding in problem – solving was same as illustrated by Table 13. There was no significant means difference between control and experimental groups. The slight mean difference was statistically insignificant at $\alpha = 0.05$ as $t = 0.174$ with small effect size of $d_s = 0.17$ from Cohen's power test interpretation. This has shown that groups were of same strength in terms of performance of Mathematics achievement test before intervention initiated.

Hypothesis stated that there was no significant difference in mean scores on effects in students' performance scores on those taught using problem–solving approach and others taught using convention methods retained. The findings agreed with Njoroge & Githua (2013). In their study, they found out that there was no statistical significance between experimental and control groups' difference in pre-test Mathematics Achievement Test before commencement of intervention on cooperative learning strategy.

Treatment on experimental groups E1 and E2 commenced, where students taught using problem–solving technique whereas control groups C1 and C2 taught using conventional methods. Solomon Four Group designs employed to study experimental group's experiences intervention prior to post-test measure affected on students' Mathematics performance. Activities involving students had background from basic concepts developed from advanced ideas on topics.

The questionnaire of post-test administered to all participants in their respective categories of schools immediately after treatment. The twenty post-test items composed based on comparable topics in pre-test. The topics discussed during intervention applying problem–solving as well as conventional instructional methods. Post – test analysis of post-test C2 and pre-tests E1 and C1 results tabulated in table 14 to prove resemblance of students' pre-test awareness.

Table 14: Pre-test E1 & C1 and Post – test C2 Performance per Group

Combined group	NUMBER	MEAN	STDEV	Standard error	95% C -I
E1 pre-test	128	39.36	16.39	1.454	36.52 - 42.20
C1 pre-test	147	36.57	15.79	1.307	34.0 – 39.12
C2 post-test	143	34.92	14.53	1.219	32.54 - 37.30
Combined	**418**	**36.95**	**15.40**		

Source: Murang'a County Field data, (2018)

Table 14 displays those students who took part in experimental E1, and control C1 groups pretests have same performance as posttest obtained by control group C2. This divulged claim that control group C2 has same mean at posttest although it not sensitized with pretest as experimental group E1 and control C1 group. The pre-test of E1 and C1 compared with post – test of C2 indicated on table 14 had zero or near zero effect signifying that groups were similar without intervention.

Hypothesis tested was no significant mean difference between achievement of control C1, and experimental E1 groups in pretest and posttest of group C2 i.e., C2 had no effect intervention.

To test this, hypothesis ANOVA applied. The null hypothesis is vital in testing the differences in more than two normally distributed populations. S^2_B means variations between the means, which implies variations of the group means about the overall mean, GM and S^2_W variations within group variations. This was where variations of the individual values about group means. Table 15 is the ANOVA table showing that groups in study have a common entry point at any first test.

Table 15: ANOVA Pre-test E1 and C1 and Post – test C2 Performance on Three Groups

	Variation	Df	Mean square	$F = \frac{S_B^2}{S_W^2}$	F - value
Between groups	676.98	2	1.62	2.796	3.02
Within groups	242.11	415	0.583		
Total	245.34				

Source: Murang'a County Field data (2018)

Table 15 gives mean scores of variation of students' Mathematics performance in pre-test of groups E1 and C1 pre-test and post – test C2. The ANOVA results for F – test pretest of E1 and C1 and C2 post – test do not have difference despite one group having pre-test.

> *Null hypothesis was accepted that no significant variance difference on students' Mathematics performance before problem – solving was applied and after application on control groups.*

There was enough evidence that three mean scores had no difference between the groups before any treatment done. The critical value at F (2, 415) was 3.018, whereas tabulated F = 0.014, (2.796 < 3.018). The null hypothesis accepted that means are the same for both experimental E1, and control C1 groups before intervention is the same as control group C2 after intervention.

> *Hypothesis tested that there was no significant distinction mean difference between achievement of the control C1, and experimental group E1, in pretest and posttest C2.*

The results from ANOVA for pre-test E1 and C1 in Solomon Four Group confirms that C2 post – test do not have any difference after intervention of experimental groups.

Scheffe's post hoc comparison test on the hypothesis H_{02} carried out. The results were F = 2.789 to accept the null hypothesis then comparing E1 and C1 gives 2.20, E1 and C2, results 2.35 while C1 versus C2 gives 0.82. All three comparisons revealed that they were less than critical point showing that students had no effect if problem – solving not applied in teaching procedures.

Mean scores of students' Mathematics performance in the post-test of groups E1 and C1 results shown in table 16.

Table 16: Statistics for Mathematics Performance Assessment Paired Differences Post-test Experimental Group E1 and Control Group C1

Variable	Mean	Standard deviation	Standard error	95% confidence interval		t	Degree of freedom
				Lower	Upper		
PosttestE1	48.91	15.42	1.368	46.24	51.58	5.63	274
PosttestC1	38.15	16.26	1.346	35.52	40.78		
Combined	**43.53**	**15.82**	**1.245**				
Difference	10.76						

Source: Murang'a County Field data (2018)

Table 16 expresses those effects of students' Mathematics performance after treatment of experimental, E1 group and control C1. These groups received pre-test before intervention commenced. The two groups received post-test immediately after intervention to experimental groups. The results were recorded as Pr (|T| > t) = 0.68, with positive moderate Cohen's d effect 0.68 which is a reasonable improvement.

Hypothesis tested was that there is no statistically significant mean difference between achievements of control C1, and experimental E1, group in post-test.

Hypothesis claimed that there no significant means difference in Mathematics performance between students taught using problem–solving approach against those taught using conventional strategies in secondary schools in Murang'a County. Table 16 further has shown that there was significant improvement in students taught using problem – solving in performance after intervention period. It was obvious that problem – solving enhanced students' performance in Mathematics.

Students' changed behavior in Mathematics classroom observation rating scales on problem – solving showed that students' achievement improved. During experimental, observation had intension of demonstrating results of average ability students could read problems more carefully throughout intervention period. When problem – solving was used different strategies applied with purpose of increasing perseverance in problems and checking of solutions. During the intervention period, learners tried problems better and better getting closer to the solution while peer partners gave her/him opportunities to make out different ways to approach mathematical problems.

The hypothesis rejected that the mean difference scores during post-test for both experimental E1 group and control C1 group were same. There was significant difference between experimental and control group, since 5.63 > 1.967 (p-value is small 0.0003). Mean score of experimental groups significantly higher compared with that of control group at post-test. Cohen's d was 0.68 which illustrated that there was moderate positive effect size of the means difference on experimental E1 and control group C1.

The two groups had experienced pre-test. The implication was that significant positive improvement on students' achievement in Mathematics taught using problem – solving. These results revealed that both groups experienced post-test difference was due to intervention. Problem–solving approach enabled students in restructuring the problem through better understanding of problem. The social interaction in the classroom community helps to promote motivation in self-driven procedures to construct reasonable solutions to the problem. The time taken in preparation was useful for improvement on Mathematics achievement. This improvement supported by Nu Nyunt,

(2009) who claimed that problem – solving improves students Mathematics achievement by using classroom interaction environment where students assist one another.

The results show the Experimental group E1 which experienced pre-test and control group C2, which did not experience pre-test nor intervention. This was predicting whether intervention had any effect during post – test whereas control did not experience pre-test. The results on students Mathematics performance post – test assessment between experimental E1 and C2 shown in table 17.

Table 17: Statistics for Mathematics Achievement Assessment Paired Differences Post Test Experimental Group E1 and Control Group C2.

Variable	Mean	Standard deviation	Standard error	95% confidence interval		t	Degree of freedom
				Lower	Upper		
PosttestE1	48.91	15.42	1.368	46.24	51.58	T =7.66	269
PosttestC2	34.92	14.53	1.219	32.54	37.30		
Combined	41.92	14.90					
Difference	13.99						

Source: Murang'a County Field data (2018)

Table 17 gives an idea on post-test with the aim to show experimental group E1 and control C2 differ significantly in their mean scores. The consequences on the two groups had shown marvelous difference in mean scores.

The following hypothesis was tested.

There was no statistically significant difference in mean scores between controls C2, and experimental E1, groups in post – test on students taught using problem–solving strategies with those taught by means of conventional methods on Mathematics achievement in secondary schools.

The analyses of post – test results illustrated in table 17 demonstrating that statistical difference in mean scores students' performance on those taught using conventional methods had significant effect on those facilitated using problem – solving in secondary schools in Murang'a County. The two groups E1 and C2 at $\alpha > 0.05$ and $t = 7.66$. This was clear evidence that problem–solving improved students' performance where $7.66 > 1.96$, null hypothesis rejected. The Cohen's d of 0.94 implies that there was large positive effect size of means difference and p-value was too small.

A number of participants in E1 solved the problems in post – test showing good understanding of question better than, those in C2 participants whose score is lower. This attributed to change in knowledge through the duration of intervention. The concept of problem – solving applied during intervention made students advance in understanding the problems better. The careful analysis of key complex information reducing it to smaller thoughts provided way to attain reasonable solution. The brainstorming in-group participation and working together improves mathematical communication. The skills developed in this class were relevant to real – world where working environment was expanded to know how to think logically and critically.

Two groups E1 and E2 were experimental whom experienced intervention although E2 did not receive pre-test. These groups commenced treatment at same time although E2 involvements in pretest was limited. Students were engaged in problem – solving; not only using worksheet intervening questions and guidance but they were encouraged to interrogate and reorganize encompassed problems applying familiar situations. This becomes a vehicle for learners to develop problem–solving techniques of construction, evaluation and redefining their own Mathematics theories (Crebert et.al, 2011). Further characteristic to show problem–solving approach give confidences student to making their own simplification of concepts and rules in which Mathematics learning centered.

The results on students Mathematics performance posttest assessment connecting experimental E1 and E2 shown in table 18.

Table 18: Statistics for Mathematics Achievement Assessment Paired Differences Post – Test Experimental Group E1 and Experimental Group E2

Variable	Mean	STDEV	Standard error	95% confidence interval		t	Degree of freedom
				Lower	Upper		
Posttest:E1	48.91	15.42	1.368	46.24	51.58		127
Posttest:E2	45.80	14.24	1.274	43.31	48.29		125
Combined	**47.37**	**14.90**	**0.937**	**45.54**	**49.20**	**1.67**	**252**
Difference	3.11						

Source: Murang’a County Field data (2018)

Problem–solving teaching approach used during intervention seeking improvement of student’s achievement in Mathematics. Second objective achievement tested by means of following null hypothesis.

> *Null hypothesis was that insignificant difference in mean scores between two groups E1 and E2 those, which underwent treatment in students’ Mathematics achievement after post-test.*

The critical value at 95% confidence limit in degree of freedom DF = 252 is 1.967 by interpolation. When t = 1.67 (1.67 < 1.9670), the hypothesis was accepted that there was enough evidence that means do not have any substantial difference. The Cohen’s d of 0.21 which has shown that there was small effect size of the means difference which might have been affected by pre-test on E1 cluster schools. The effect was close to zero concluding that both groups have equal mathematical knowledge after intervention.

The two groups C1 and C2 were control groups, and they did not experience intervention although C1 did receive pre-test. The results on students Mathematics performance post – test assessment between control groups C1 and C2 shown in table 19.

Table 19: Statistics for Mathematics Achievement Assessment Paired Differences Post – Test Control Group C1 and Control Group C2

Variable	Mean	Standard deviation	Standard error	95% confidence interval		t	Degree of freedom
				Lower	Upper		
Posttest C1	38.15	16.26	1.346	35.52	40.78	1.775	288
Posttest C2	34.92	14.53	1.219	32.54	37.30		
Combined	**36.26**	**15.49**	**0.911**	**34.48**	**38.04**		
Difference	3.23						

Source: Murang'a County Field data (2018)

Table 19 shows the results of achievement in post – test administered to control C1 and control C2 groups. These groups did not go through any treatment at same time, then C1 received pre-test, whereas C2 did not take pre-test.

Hypothesis that there was statistical insignificant difference between scores of C1 and C2 on students' performance taught using conventional methods in secondary schools in Murang'a County.

The results revealed that there was no significant mean difference in students' Mathematics achievement after post-test when two control groups did not undergo treatment. The critical value t at 95% confidence limit in degree of freedom DF = 288 was 1.968 by interpolation. When t = 1.775 (1.775 < 1.968), the hypothesis was accepted and there was enough evidence that mean scores did not have any statistically significant difference. The Cohen's d of 0.21 which shows that there was small effect size or near proving that the means difference has not been affected.

Table 20 displays achievement of combined experimental groups' post-test and combined control groups post – test total means. Tables 18 and table 19 have already given an idea that there were no significant means scores difference between both experimental groups E1 and E2 and both control groups C1 and C2 together. Table 20 now presents combined data for both experimental and control groups. Table 19 showed that control groups C1 and C2 have t - value 1.775, which was less than critical 1.96 at 95% confidence interval. As t – score is within this value, there was nothing to suggest that there was any difference between the two means and hypothesis was accepted.

Table 20: Mathematics Achievement Post–Test for Combined Experimental and Control Groups

Variables	No of respondents	Mean score	Standard deviation	Standard error	95% confidence interval	
					Lower	Upper
Post – test E1	128	48.91	15.42	1.368	46.24	51.58
Post – test E2	126	45.80	14.24	1.274	43.31	48.29
Combined	**254**	**47.37**	**14.90**	**0.937**	**45.54**	**49.20**
Difference		**3.11**			**-2.46**	**2.70**
Post – test C1	147	38.15	16.26	1.346	35.52	40.78
Post – test C2	143	34.92	14.53	1.219	32.54	37.30
Combined	**290**	**36.56**	**15.49**	**0.911**	**34.48**	**38.04**
Difference		**3.23**			**-2.42**	**2.59**

Source: Murang'a County Field data (2018)

Table 20 indicates a difference between mean scores of experimental groups E1 and E2 and control groups C1 and C2 with intention, showing post – test found to be significant at 0.05 levels. When two experimental E1and E2 combined, and two control C1 and C2 combined yielded Cohen's d 0.71, which had moderate positive effect. Hence, rejecting null hypothesis:

Null hypothesis there was no significant difference among mean scores of combined experimental group and control group during post –test.

There was no significant difference among mean scores of collectively associated with experimental groups and control groups during post – test shows evidence of rejecting null hypothesis. The significant impact of treatment had found on achievements of Mathematics students taught through problem – solving rather than those taught in conventional methods. Clearly, an indication that experimental groups E1 and E2 who were exposed to problem–solving strategies, have shown better Mathematics performance than control groups C1 and C2 respectively. The implication here was that when learning Mathematics through problem–solving strategies, students' performance increased. This was supported by results of Lesh and Zawojewski, (2007) observation that engaging students in mathematical problem–solving activities would help them discover mathematical ideas, which are a necessary tool for problem – solving.

This study took ten weeks, which was reasonable period to observe that problem – solving affects students Mathematics learning outcomes. The study investigated the difference between students taught through problem– solving and those taught using conventional approach. Problem–solving approach used in this study applying constructivism where students construct knowledge through their experiences. Students were actively involved increasing enjoyment and social skills in communication. Problem – solving provided an opportunity for young mathematicians to explore ideas to improve their achievement.

Students' Mathematics achievement enhanced in each category during post – test as shown in Table 21.

Table 21: Post – Test Performance per Category

	Experimental E1 and E2 combined				Control C1 and C2 combined			
Category	**NO.**	**Mean**	**STDEV**	**C.I.**	**NO**	**Mean**	**STDEV**	**C.I**
A	66	61.14	8.66	59.05-63.23	90	46.60	11.2	44.29-48.91
B	62	54.46	9.58	52.07- 56.84	61	43.31	9.53	40.92– 45.70
C	72	38.42	9.64	36.19- 40.65	77	31.63	10.87	29.20–34. 06
D	54	34.32	14.14	30.55- 38.09	62	28.67	12.32	15.60 -21. 74
Combined	**254**	**47.37**	**14.90**	**45.54- 49.20**	**290**	**36.56**	**15.49**	**34.78– 38.34**

Source: Murang'a County Field data (2018)

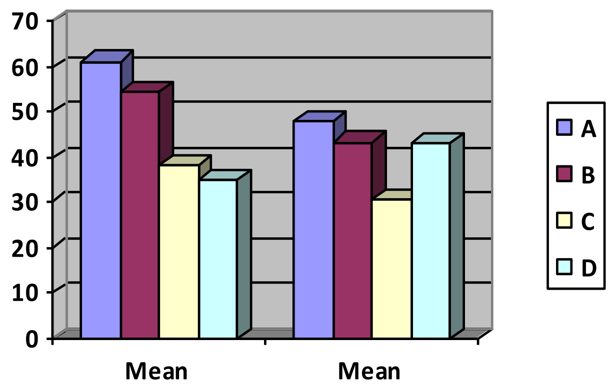


Figure 6: Category Comparisons Experimental E1 & E2 and Control C1 & C2

Table 21 shows that students' who presumed to be of low ability, through the study, revealed significant gains in Mathematics achievement after intervention of E1 & E2. The control groups C1 & C2 did not improve in the Mathematics achievement. The effect size was Cohen's d equal to 0.71, which was a moderate positive effect in combined groups. The slight difference between E1 post-test and E2 post-test category of secondary schools explains the effect that pre-test has had upon treatment. Further, it shows that statistical mean score differences between the experimental groups E1 and

E2, control groups C1 and C2 were insignificant. There was enough evidence to accept null hypothesis that mean scores between control groups and experimental groups were significant.

Figure 6 reveals that each category improved after intervention despite the ability of students. The bar graph shows almost a similar increment spread throughout the categories. These had shown that teaching Mathematics through problem – solving on student achievement had positive effect. The respondents of control groups scored less mean score compared with experimental groups. The importance of these statistics is to show pictorial perspective of each category.

There was insignificant difference between performance experimental category groups E1 and E2, although E1 did pre-test. Similarly, the research determined that there was no significant difference between category control groups C1 and C2 without any effect of pre-test. Study found that classrooms community and culture promoted among students Mathematics achievement in all types of schools. Findings conclude that there was significant improvement on students' Mathematics achievement between those who used problems - solving versus those who practiced conventional strategies. The decision of comparing means scores of experimental and control groups proved the supremacy of problem – solving over conventional methods. These demonstrated successful application of problem – solving as an advanced pedagogical skills of teaching Mathematics.

Table 22 shows Solomon Four Group pre-test and post-test performance comparison in their respective groups E1, E2, C1 and C2.

Table 22: Solomon Four Group Pre-test and Post-test Comparison Performance

	Number	Mean
Pre-test experimental E1	128	39.36
Post-test experimental E1	128	48.91
Pre-test control C1	147	36.57
Post – test control C1	147	38.15
Post – test experimental E2	126	45.80
Post – test control C2	143	34.92

Source: Murang'a County Field data (2018)

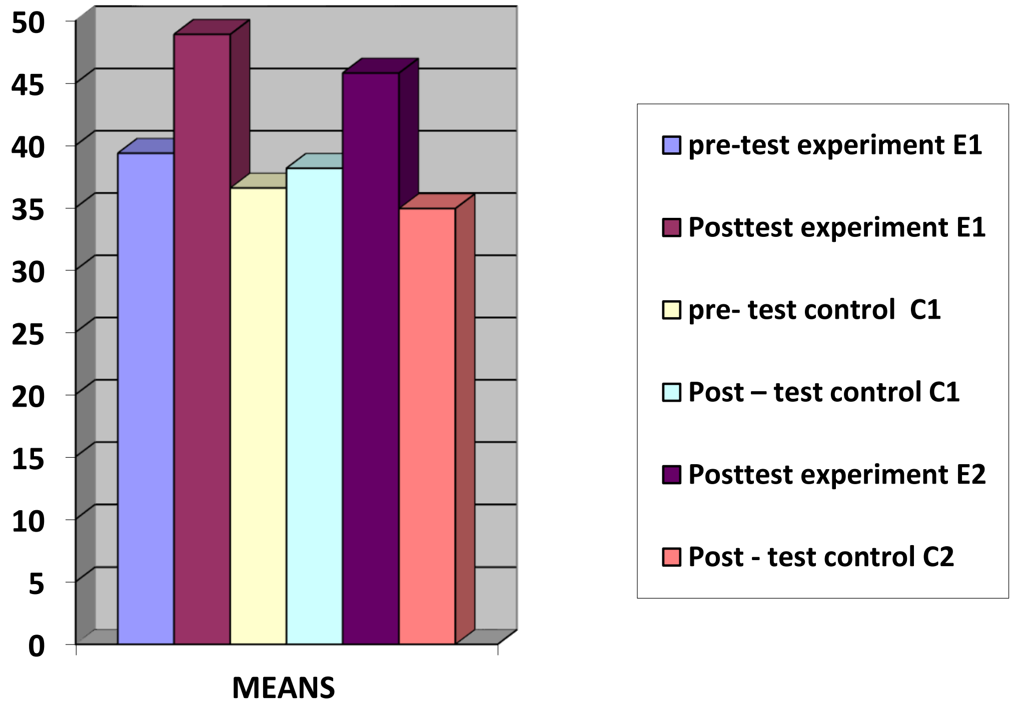


Figure 7: Comparisons Experimental E1 & E2 pre –test and post-tests and Control C1 & C2 pre-test and post –test

Figure 7 presents availability pre-test findings that had affected learning and teaching Mathematics in secondary schools. After intervention in both experimental (E1 and E2) groups, reasonable improvement in Mathematics achievement against control (C1 and C2) groups shown. Experimental groups and control groups comparison confirms that improvement of Mathematics achievement occurred in those secondary schools which had intervention and those schools which did not have intervention. The importance of teaching Mathematics through problem – solving had positive result on student achievement as it revealed by Figure 7.

The research finding results of all four groups shown by Table 23 post-test performance per group.

Table 23: Post – Test Performance per Group

Combined group	**Number**	**Mean**	**Standard deviation**	**Standard error**	**95% C -I**	**T – value**
E1	128	48.91	15.42	1.368	46.24-51.58	
C1	147	38.15	16.26	1.346	35.52-40.78	
E2	126	45.80	14.24	1.274	43.31-48.29	
C2	143	34.92	14.53	1.219	32.54-37.30	
Total	**544**					

Source: Murang'a County Field data (2018)

Table 23 shows ANOVA hypothesis testing applied, which revealed the tabulated values in ANOVA. These results analyzed using ANOVA to investigate the null hypothesis.

> *Hypothesis that there was no statistically significant difference between means of post-test scores on students' achievement in Mathematics while taught through problem–solving strategy in experimental group against those in control group taught using conventional methods.*

The hypothesis tested was insignificant differences between post-test experimental groups mean scores and the post-test control groups mean scores, with (t = 1.967) and (t

= 1.96) and small effect size of f = 0.18. The post-test items developed on higher order thinking skills that required students to reason before applying procedure for getting solutions. The assessment involving synthesis and evaluation levels as classified by Blooms (1956) taxonomy of cognitive objectives categorization. This shows that a problem–solving strategies Mathematics teaching and learning can assist students to reason and help them develop creative and critical thinking. Students improves in Mathematics by understanding of concepts, hence, achieve better performance.

Students' achievements significantly improved when teachers were aware of how to help students construct knowledge by applying problem – solving in Mathematics. Students' understanding of unfamiliar situations was overwhelmed through intuitive solution methods students used when solving problems. Teachers utilized knowledge of classroom environment when planning and conducting instruction in Mathematics allowing students' collaborative interactions.

Table 24 gives result of ANOVA of the means difference in the post-test scores combining all strata in their respective groups.

Table 24: ANOVA Post – test Performance on Four Group Means Difference

	Number	Mean	Variance	Between groups	Within groups
E1	128	48.91	237.72	583.17	229.82
C1	147	38.15	264.39		
E2	126	45.80	202.80		
C2	143	34.92	211.03		
Combined	544	41.95			

Source: Murang'a County Field data (2018)

S_B^2 : The Between Group Variations

S_W^2 : The Within Group Variations

Table 24 demonstrates that significant mean differences in student performance in post-test between all four groups caused by intersection of treatment. The second null hypothesis then concluded.

> *Testing hypothesis two (H_{02}) which states that, there was no statistically significant mean scores differences between Mathematics students' achievement in secondary schools of Murang'a County of those taught through problem–solving approach in contrast to those taught using conventional strategies during post – test results.*

Analysis of variance (ANOVA) was carried out to post – test mean scores on students' Mathematics achievement revealing; $F (540) = 2.537$, $p = 0.01$, $\alpha = 0.05$ where $p < 0.05$. The study concluded that problem – solving employed in learning Mathematics increases students' achievement (performance) rather than instructing making use of conventional approach. Therefore, hypothesis (H_{02}) which was stated students' performance in Mathematics when taught using problem – solving approach was better than students' taught using conventional strategies during post-test in secondary schools in Murang'a County.

A quasi – experimental involving social interaction process developed positive effect showing change in performance and attitude of learners applying problem–solving strategies during treatment. Scheffe's post hoc test done since there was interest in the mean difference between experimental and control groups irrespective of pretest experience spread over specific groups. The test compares two means at a time using possible combinatorial of means. The critical value F' = 12.84 for F (3, 540) compares with E1 versus E2 = 2.672 and C1 versus C2 = 3.29 revealed that experimental and control groups themselves had insignificant difference. The combinations E1 versus C1 = 34.47, E1 versus C2 = 57.52, E2 versus C1 = 17.78, and E2 versus C2 = 35.20 which were greater than critical value F'. The conclusion that there were significant differences in performance of experimental and control groups showing that intervention on teaching using problem – solving critically improves students' achievement.

Therefore, hypothesis rejected since this result was statistically significant. There was enough evidence to reject the claim since groups, which received pre-test and those given post – test only shows that intervention had been effective. The acceptance that teaching through problem–solving improves student Mathematics achievement have different opinion with Kirtikar (2013) who reacted on the criticism that conventional teaching triggers critical thinking in classroom discussions. The cognitive skills and holistic learning environment for students through problem – solving could be encouraged in secondary schools Mathematics teaching in Murang'a County. The study has given an idea that teaching Mathematics through problem–solving developed better Mathematics achievement.

In conclusion, Masingila, Lester and Raymond, 2011 postulate that many teachers give up in the process of problem – solving. They established that the role of facilitating problem – solving has been challenging due to time consuming and syllabus coverage. The study endorsed that transmitting mathematical ideas through problem – solving is more effective. The teacher should create an enabling environment through which the learner is stimulated to construct his or her own conceptual knowledge. The organized classroom activities facilitate successful problem–solving approach. The conventional methods of learning Mathematics could be more effective when seriously exploited allowing growth of students' skills through problem–solving proficiency. A profound understanding by teachers on problem–solving strategies causes improvement of students' performance of mathematical tasks.

4.2.3 Students Attitude towards Mathematics when Taught through Problem–solving

Third objective was to assess students change in attitude toward Mathematics when imparted through problem–solving approach with those taught by means of conventional methods. The present study compared attitudes of students towards Mathematics when taught with problem–solving (experimental group) besides those taught using conventional methods (control group) at post – test level. Summary mean scores and standard deviation before pre-test and results of attitudes towards Mathematics before intervention given in table 25. Pre-test results as shown for all categories participating at experimental group E1 and control group C1 analyzed.

Table 25: Two – Sample t –test with Equal Variances on Mean Scores of the Pre – Test for Experimental E1 and Control Group C1

Variable	No of respondent	Mean score	Standard deviation	Standard error	95% confidence interval	
					Upper	Lower
Pre –test E1	128	45.6	3.04	0.19	45. 07	46.13
Pre –test C1	147	45.52	4.07	0.30	44.86	46.18
Combined	275	36.99	3.61	0.17	36.56	37.42
Difference		**0.08**			**-0.5**	**0.68**

Source: Murang'a County Field data (2018)

Table 25 exposes results of attitude test before intervention. The objective of the study was to compare achievement of students taught Mathematics by problem–solving method and those taught by means of conventional methods.

The hypothesis tested was that there was no significant mean difference between experimental group (E1) and control group (C1) in attitude at pretest.

Table 25 reveals that there was insignificant statistical difference in mean scores of both groups. Data had been tabulated to $t = 0.16$ value at $\alpha = 0.05$ level. This shows that student's attitude was on the same level on pretest. Hence, null hypothesis accepted that both groups have equal Mathematics attitude on pretest. Okigbo and Osuafar when working on Mathematics achievement declared that students have same knowledge and attitude before commencement of the experiment.

The attitude questions distributed into three sub- scales. These scales were for monitoring students' attitude towards learning and teaching Mathematics in secondary schools. These subscales included Mathematics behaviour (MB), Mathematics confidence (MC) and Mathematics Engagement (ME). The elaborative feedback on treatment of experimental and control groups given by computing experiences separately instead of giving total score. The post-test attitude reported as individual item rather than aggregate results (Ross & Morrison, 2002).

Table 26 shows post-test results according to individual items MB, MC and ME in percentage scores.

Table 26: Posttest Comparison Attitude of Experimental Groups E1 & E2 and Control C1 &C2

	Number	Percentage
Experimental MB	254	24.5%
Experimental MC	254	34.5%
Experimental ME	254	21.5%
Control MB	290	20.5%
Control MC	290	27%
Control ME	290	18%

Source: Murang'a County Field data (2018)

The students interacted and discussed during intervention, so they gained confidence, changed behavior and they engaged in Mathematics. There was significant improvement in attitudes towards Mathematics and students' achievement when taught through problem – solving. This agreed with Dutton who claimed that most standardized achievement tests in Mathematics were obtained in subtests in attitude on problem – solving. The results shown in table 26 reveals that Mathematical attitudes was promoted through problem – solving. Attitude assessment towards Mathematics revealed that experimental groups after post-test at 80.5% against control groups at 65.5%.

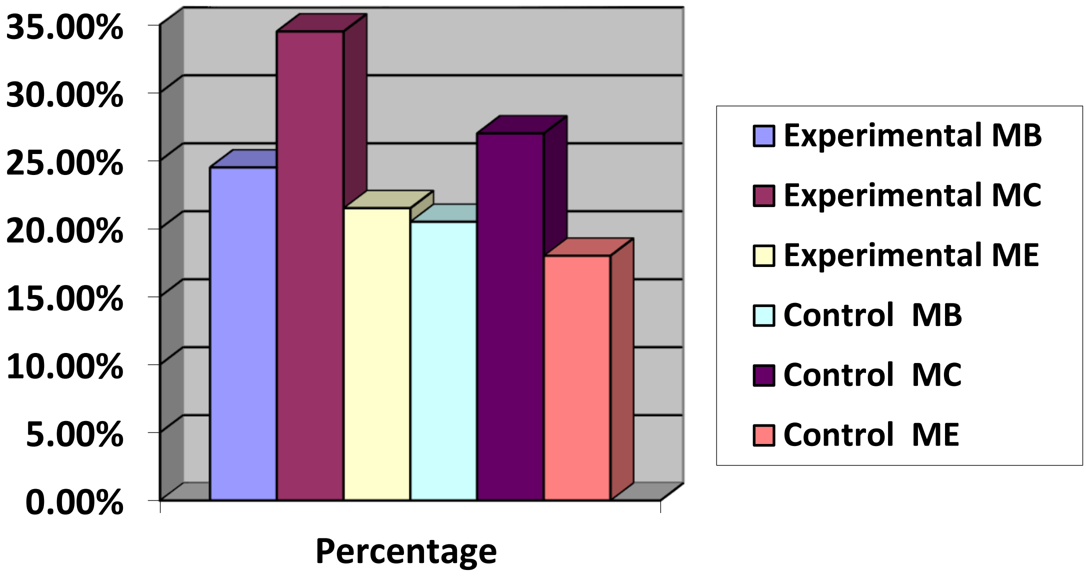


Figure 8: Students' Attitude Mathematics Behavior, Confidence and Engagement

Figure 8 chart shows that experimental components were all better than in control groups after intervention. Mathematics behavior included students' reaction on the subject while Mathematics confidence increased. The activities during intervention improved learners' engagement.

The third objective was students' change in attitude towards Mathematics taught using problem – solving weighing in contrast to those taught using conventional methods on Mathematics achievement in public secondary schools. The role-played by means of students' attitude aligned with third hypothesis.

> *Hypothesis states that mean difference between learners' attitude towards Mathematics when students taught using problem–solving approach compared to those taught using conventional methods in Secondary schools in Murang'a County is statistically insignificant.*

Paired student t – test was applied to test H_{03} in which pooled combined mean and standard deviations were used. Table 27 shows combined means and pooled standard deviations of E1 and E2 as well as C1 and C2 post intervention.

Table 27: Paired t –test of Combined Mean Scores and Pooled Standard Deviation for Experimental E1 and E2, together with Control Group C1 and C2

Variable	No of respondent	Mean score	Pooled standard deviation	95% confidence interval	
				Lower	Upper
Posttest: E1& E2	254	57.6	12.01	56.12	59.08
Posttest: C1&C2	290	46.4	13.37	44.85	49.94
Combined	544	52	12.73	50.93	53.07
Difference		**11.2**		**- 0.60**	**2.42**

Source: Murang'a County Field data (2018)

Table 27 has shown there was no statistical difference between post– test experimental groups (E1 & E2) combined mean score and post – test control group (C1 & C2) combined mean score and their respective pooled standard deviations. The value t = 0.88, p = 0.0146 at α = 0.05 with Cohen's d power effect of 0.88 which was large positive result, this shows that the attitude of the students was not on the same level after intervention. Null hypothesis was rejected in preference of the claim that students' attitude changed. Then accepting alternative hypothesis that there was statistical significance in attitude towards Mathematics on students taught using problem–solving have positive attitudes than students taught using conventional strategies.

4.2.4 Prototype Lesson Plan

Fourth objective was developing a prototype lesson plan, model of teaching Mathematics in secondary schools of Murang'a County applying problem – solving, which was developed. Problem–solving model lesson plan consist of structural systematic approach. This was first to ensure that there was consistency in students' participation in classroom discourse. The teachers and students are required to know what process others are using to keep process more scientific. Second, the model provides focus for group activities where individual contribution in making decision to arrive at consensus is paramount. Group dynamics considered as promising leading to solving problem to obtain all possible solutions. The Model uses series of logical phases to facilitate individual student as well as groups in identifying most important courses of best solutions.

Thirdly, the model provided avenues to examine all steps and eliminate those, which cannot work in each situation. The Model presents process improvement activities to arrive at a well thought solution. The model developed using Bloom's cognitive taxonomy domain. The steps are as follows:

Step one: The learner has acquired knowledge to enable her/him to define the problem. The problem diagnosed and redefined according to student experiences. If the concept is not familiar, it will be the role of a teacher to guide students to improve understanding.

Step two: The student determines the root causes of problems related to existing concept. The concepts familiar to student are required to comprehend assumed situations in which students are working. These concepts are useful in synthesizing information to what he\she has learned and apply it to new situation.

Step three: The student generates solutions, which related to the concept. Students can develop alternative solutions and attempt to work the problem backward. The algorithm followed, and a devised plan used to explore further through students' interactions.

Step four: The student select solution, which is acceptable. The heuristics plan of carrying out problem – solving by selecting suitable strategies. At this stage, the learner focuses on construction of mathematical knowledge through social interaction and conceptual understanding.

This researcher developed model lesson plan used to demonstrate to teachers that problem–solving strategy improves learners' resourceful thinking and creativity. The development of problem–solving strategy puts teachers and students focused on the skills, which made them active participants in learning process. The use of technology should complement learning Mathematics through problem – solving. Mathematics serve as foundation knowledge which changes learners' mind set by shaping their attitude. This plays an important role in provision of life – long education for SDG. Problem – solving is one of competencies that students must possess in learning Mathematics in public secondary schools in Murang'a County. Competence of solving problems is an expected product that student is enabled to build new knowledge of Mathematics by social constructivism.

The research maintains that a good lesson plan for mathematical instruction must present Mathematics as a problem – solving as a continuous process of investigation which remain open to review. This contrasts current conventional methods which emphasis on content approach through mastery of rules and procedures. There is believe that students' involvement in construction of knowledge emphasis on improved performance and attainment of good performance.

The lesson plan provides problem–solving approach that favors student-centered instructional method. This gives students' personalized instructions perking up their interests and motivation. Problem–solving approach will yield better performance in Mathematics. The reliable differences in results could attribute to collaboration in classroom.

Secondary school Mathematics usually positioned as a difficult discipline by students in relation to other disciplines. Current study suggest that students and teachers must form Mathematics communities whose focus stands to give accidental experience on how to do Mathematics. These communities are useful in socio – cultural environment where students enabled to manage mathematical activities in formal and informal situations. Socio – cultural applies social constructivism theory, which views Mathematics learning as having crucial implications in promotion of problem – solving. Teachers and students in socio – cultural environments were careful expert participants who were fully involved in the discourse, standards and practices of classrooms.

LESSON PLAN FORMAT

Lesson title:

Form:

Subject Area: Brief content explanation

Essential question(s): Establish content to the students' prior knowledge

Instructional Objectives: SMART

Time bound	Descriptive	Guided practice	Independent & groups discussion
Introduction (5 min)	Revisit Mathematics ideas and strategies from previous lesson(s) that relate to learning objective(s). Including a strong motivational device to connect to the lesson activities.	This is a process to engage student to activities, which related to objective(s) depending on students' experiences. Therefore , connection to prior learning, motivational device relationship to real life experiences and relevant to future learning	learner dialogue to assist each other understand the problem
Teaching process (25 min)	During this phase learning how to solve problem commence individually, in pairs and small group.	Step – by – step processes and procedures using Polya theory steps of understanding, device plan, carry out the plan and look back. Learning activities include process, procedures and strategies that support lesson discussed by students. The teacher may support these processes through guided practice, which explicitly and sequentially described. Students are also given an opportunity to practice and apply the skills under teacher's supervision	Objectives focus function on the work done individually and in small groups to a whole class discussion. Logical order progress evident student learning.
Consolidation (10 min)	Whole class discussion and analysis of students' solutions, highlight Mathematics ideas and practise by	The lesson designed bring a students' presentation to an appropriate conclusion as they bring their own minds to use, apply and	Encourages higher thinking and helping students conceptualize

	solving similar problems	extend through reflection	theme of the lesson. Challenge students to think critically and mode their ideas.
Purpose	Problem–solving	Aim of problem – solving is to develop effective mathematical communication using high thinking skills analysis, synthesis, and evaluation.	Methods of formative and summative assessment are established.

Instructional resource materials and technology applied effectively on the lesson engaging students hence provide for maximum student learning. Resources integrated into the context of content taught. These resources and technologically utilized to support instructional objectives.

The lesson plan is concluded by noting that students are guided to practice five practices to improve their Mathematics problem – solving. These practices are anticipating, monitoring, selecting, sequencing and connecting. The lesson geared towards providing skills to stimulate student thinking following two categories. First, lesson plan facilitators interrogate student in order to help them illustrate and identify the level of the problem. These guides' learners to think deeply while facilitators ask probing questions sequentially to explore meaning and relationship of Mathematics development. The student is encouraged to explain their way of thinking in so doing help them discover the methods of mathematical reasoning which is central to problem – solving.

Second is motivating students to think positively through participating in social interactions. When they make good use of environments, students are stimulated to decompose complex problem into smaller splits. Decomposing problem into partial problems catalyses increase of imagination and reasoning. In classroom students attains skills of listening, questioning to make sense and building one another practices as teachers can develop. Teachers keep asking questions to promote class discourse by grasping students' attention and accountability for their active engagement.

Traditionally lesson plans considered as directions for implementing particular lessons with emphasis of procedures and structures. This promotes algorithmic thinking and inadequately employing practices, which turn off student public communication. These arrangements have limited attentions on how lesson will aid students to develop key mathematical idea understanding.

The following problem – solving was adopted from Bruner techniques of considering that learners facilitated to learning irrespective of their age. Researchers have noted that familiarity of problem structure contribute to good performance in Mathematics. This study contributes that using the five practices will improve students' positive attitude as they are motivated by including their preferences in lesson development. Problem–solving approach favors student – centeredness, which increases student interest and becomes reason for motivation of better performance.

Good problem–solving lesson plan consider structuring knowledge such that learner can grasp. Effective implementation through sequencing up-to-date material should result to generating new propositions and increasing manipulation of information. This expanded in theoretical framework to encompass social and cultural aspects of learning interactions (Vygotsky, 1978).

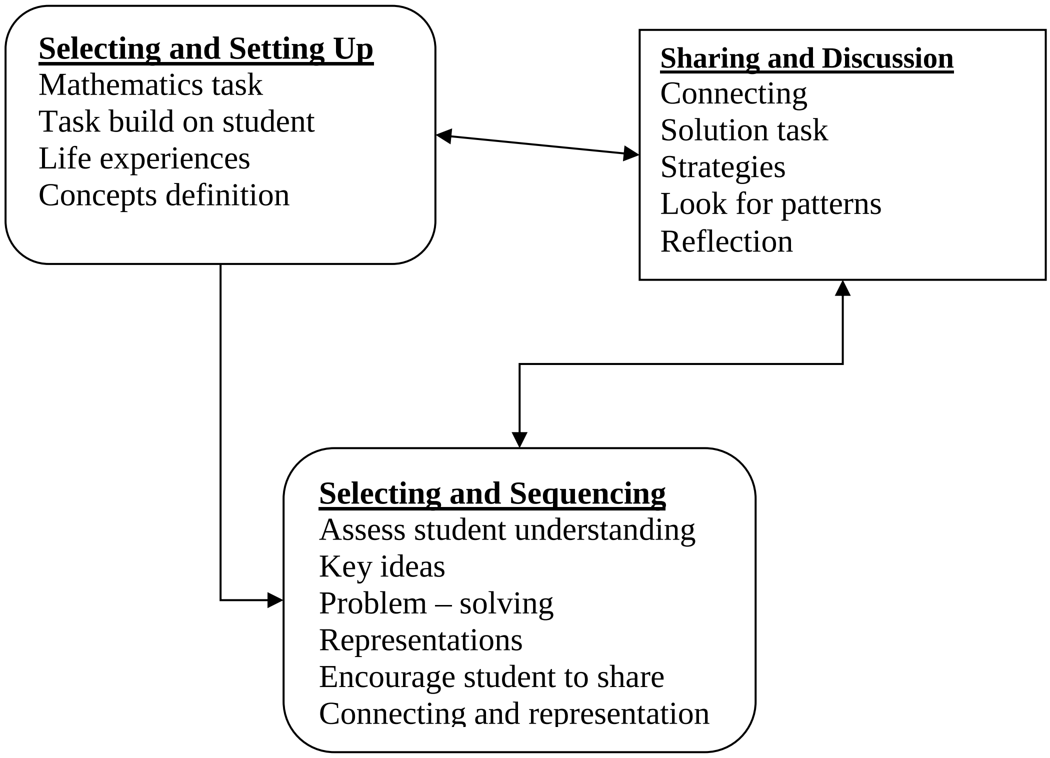


Figure 9: Problem solving cycle model

Source: Researcher (2019)

4.3 Discussion of the Findings of the Study

First objective was to establish preference of conventional strategies in teaching Mathematics over problem–solving strategies in teaching secondary schools Mathematics in Murang'a County. Accordingly, Odindo, (2013) on the study in Siaya County found that the curriculum became teacher - centered rather than learner – centered, so most of the instructors started using other conventional methods apart from problem–solving (Odindo, 2013). This study supported knowledge of social constructivism theory where students constructed knowledge in their classroom experiences rather than absorbing what they told. The constructivist views the student as an active learner who is in the process of struggling with a problem to obtain a solution. The learner usually gets deep understanding of Mathematics concepts since problem – solving provides an opportunity for students to explore ideas while given a chance to extend their creativity. The study completed a gap that exhausting problem–solving strategies improves students' achievement in Mathematics. Integrated conventional strategies in view of the fact that it becomes learner – centered rather than teacher – centered would work as well.

The second objective was to examine students' performance in Mathematics for those taught by means of solving problem enhanced by constructivism with those taught using conventional strategies in secondary schools in Murang'a County. Problem – solving was relatively a new idea in Mathematics curriculum in Kenya considering teaching as a discovery method. Typically, problem–solving approach is associated with the view that teachers' proficiency level of English language, as medium of instruction in Mathematics class is important. The mathematical modeling approach is a problem - solving as effective strategy for Mathematics education (Leiken & Rota, 2006). This study used language to support student to be able to interpret Mathematics problem and make meaningful way to devise a plan to solve it.

The teacher title role changes from designing and selecting problems to use for instructions to a guider and participant in classroom environment discourse. This is a new pedagogical trend in the 21st century Mathematics teaching which involves integrating problem – solving for competency in the classroom. Beyond promoting Mathematics problem – solving, it fosters development of valued life skills and disposition goals important for students' long-life learning. This means that students do not only use Mathematics for academic purposes, but also as future adults who can take part in challenging ventures. The teachers who participated in intervention noted that engaging students in problem–solving activities improves student perseverance, independence, critical thinking skills and general communication. The students develop a mint habit to support ones' idea or request opinion of another. This implies that they develop logical thinking, not restricted to Mathematics, but have willingness to have a dialogue. Teachers distinguished that problem solving had influenced in social relations consisted in social constructivism theory. Mathematics classroom community has impact of social dynamics whereby trust built to make out things themselves as could use their peers as resources (Lieberman & Mace, 2008).

Third objective was to assess students' change in attitude towards Mathematics when taught through problem – solving in public secondary schools in Murang'a County. Methods dealing with exposition and cooperative teaching in learning Mathematics have been studied extensively. Findings of contemporary study answers various frequently asked questions about problem–solving (Scherer & Steinbring, 2006). The study adds to literature review in learning of Mathematics through problem–solving activities. These activities done in an environment encouraging students to interact freely in order to discover concepts themselves. This improved conceptual growth, attitude change, build confidence and create a community of young Mathematicians who care for each other.

Presently, the study did detest that problem – solving is a desired teaching approach preferred rather than conventional strategies. Problem – solving is different from problem – based where given problem has algorithm to be followed to solve it. The study implied that teachers shape students' skills in problem – solving by improving instructions through proper usage of resources. Problem – solving could be used to improve students Mathematics achievement. Giving students' appropriate opportunity to be involved in problem – solving in classroom and other unfamiliar situation bring new strategies to students' attention. The teachers' attitude towards Mathematics changes through problem–solving as well as changing Mathematics from non – performance subject to enjoyment substance.

This study investigated to what extent learners are involved in learning Mathematics. This study presumed that formal learning in secondary schools in Murang'a County, Kenya, where classroom teachers with their students participate in knowledge creation in Mathematics through demonstrations of structured process and provide practices of classroom without creating caring learners' community. The study developed prototype plan to effect transformation modifications in curriculum content and instructional strategies that involves renewal effort of seeking solutions and exploring Mathematics, rather than memorizing procedures and formulae, respectively. Learning Mathematics through problem – solving ensures learners communicate conjectures instead of doing normal Mathematics exercises through which attitude towards Mathematics would be affected by extending understanding.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter precisely highlights major results of the study. Relationships of results including research studies and limitations occurring during this study. It also presents recommendations for practice including for authorities for implementation, for beneficiaries and for other stakeholders. The implications of the study relate further to future research recommendations.

5.2 Summary of the Findings

Currently the study conducted to establish purposeful effect of teaching Mathematics through problem – solving strategies on students' achievement in public secondary schools in Murang'a County. Quasi - experimental design with emphasis on Solomon Four Group factorial design followed throughout the study. This study conducted in Murang'a County, in former central Kenya region. The study sampled five hundred and forty – four (544) form three students in sixteen secondary schools. Sixteen teachers from four categories were involved for schoolwork in their respective areas. Sixteen (16) teachers including eight who trained in experimental schools to conduct intervention, while the other eight teachers remained in control schools from respective categories.

The training of teachers to help in intervention for experimental groups on new strategy lasted for a short period since most teachers already trained. The treatment tool was applied to students engaged in learning through problem – solving by creating good quality environment. These settings make students to interact in order that experimental teachers practice skills involved. The control groups taught using normal conventional approaches since they were not conscious of treatment. Data was collected using performance in pre-test and post-test, attitude questionnaires for students and Delphi questionnaires for Mathematics teachers.

The study guided by four objectives answered by evaluating three main hypotheses. Teaching and learning Mathematics concepts for understanding has been a great concern among Mathematics educators. Herbert and Wearne (2003) argued that it is not enough memorizing and drilling procedures in solving mathematical problems, but there is need to understand the process. The learner should develop the procedure him/herself for application in different situations. This helped to establish the conventional strategies used by teachers as demonstration, discussion and supervised practice as commonly used. These strategies were not learner- centered but teacher-centered since they involve teachers teaching a procedure which is followed by students.

The results found that preferences should be done for combined sampled population in the four categories with regard to approaches of their Mathematics teaching showed a positive and significant correlation at $\alpha = 0.05$ level. This indicated that all teachers agreed with practicing the teaching strategies they would prefer in teaching Mathematics. The consequences of preferences found that when teaching done through problem – solving, better students' achievement was realized. Homework, ICT, exposition and laboratory were ranked as the least used and "should be" methods. This might be because schools do not have enough facilities or teachers falsely believe that learners cannot do Mathematics. Demonstration, supervised practice, cooperative and discussion were rank as frequently used and "should be" methods.

The problem–solving strategy ranked fifth and seventh position as frequently used and "should be" respectively. The study went ahead to assess whether students' taught Mathematics through problem - solving improves their achievement. This achieved by using Solomon Four Group design showing those students using problem–solving approaches performed better than those using conventional teaching methods. Although Delphi questionnaire applied more than once to arrive at consensus answer, it became a limitation of this study. The study, therefore, had tried to address the concern of students' Mathematics achievement through teaching using problem – solving.

The second objective was to compare students' performance in Mathematics while using problem–solving rather than conventional methods. In the experimental groups whose learning mode was problem – solving, students achieved better outcomes than control groups on post – test examination. These results had shown that Mathematics imparted through problem–solving activities enhanced students' learning outcomes in relations to conceptual development. This judgment was similar to other research carried out which suggested that engaging learners in Mathematics through problem–solving actions support concepts development at high order thinking skills as disclosed by Lesh and Zawojewski (2007).

Overall mean scores on post-test achievement between experimental and control groups showed statistical difference. The treatment provided to these schools caused improvement in experimental groups, whereas these has been different from control groups although they belonged to same strata. This shows that students were capable of mastering mathematical concepts when they taught through problem–solving rather than other conventional strategies. Using different schools helped to overcome the limitation of data collection since the researcher had no control over the schools' policies and programs.

The third objective was to measure change in attitude towards Mathematics when taught through problem–solving approach. The attitude changes of students have shown statistical significance within the short period of intervention. The students who were taught using problem – solving became more confident, engaged and their mathematical behavior improved. This high mean score in attitude shows that most students from Kenya show positive attitude towards Mathematics confirmed by Odindo (2013). During the Third International Mathematics and science study, developing countries that participated in TIMSS including Morocco, South Africa, Tunisia and Philippines showed that high positive attitude means scores did not translate into high achievement in Mathematics (TIMSS, 2012).

5.3 Conclusions

The study used problem–solving approach as an effective instructional strategy to improve student Mathematics achievement by providing students with appropriate opportunities to engage freely in problem–solving activities. The study has shown that general problem–solving strategy had been successful in secondary schools' practice even in all categories. This confirmed by Lesh and Zawojewski (2007), who suggested that there was need to teach more with specific problem–solving strategies to let students effectively deploy their content knowledge in problem–solving techniques. This study differs from other research by encouraging teachers to apply constant interventions in directing students to valuable critical thinking. The 21st century pedagogical trends align teaching Mathematics through integrating problem –solving as a competency-based strategy in the classroom location (Wathall, 2016).

Peer partners and small problem–solving groups gave learners opportunities to see the different ways including teammates approach to mathematical problems. In conclusion, first, proper planned, a well - developed, and executed problem–solving instruction can significantly improve students' achievement in secondary schools Mathematics in Murang'a County. Second, an effective application of mathematical problem – solving strategies practised promote Students' interest towards Mathematics. The study concluded that a fully prepared lesson-using problem – solving strategy would be more of student - centered rather than teacher - centered. The teacher through planned intervention that involve students in problem–solving strategy, help students develop perseverance, independence, thinking skills, general communication and support other ideas hence helping them to be socially oriented. These life skills beneficial for students to take engage in challenging ventures in life – long education.

Third, problem – solving as a strategy enhances students' performance in Mathematics because there was improvement of understanding of problems in addition to smoothening Mathematics computation in classroom community. The exposure to problem–solving experiences to meaningful dialogue in Mathematics classrooms influences future social relationships. The social dynamics in their classrooms help students learn how to solve routine and non – routine problems thus improving their Mathematics achievements. This was according to Vygotsky's (1978), the theoretical framework that social interactions produce cognitive development, which takes fundamental role in human intellectual growth.

Finally, the fourth conclusion is that classroom environment provides students with variety of learning materials and culture through social context experiences. The learning materials include human resources, improvisation of materials within the environment and ability to relate content to real world creating enjoyable classrooms. Teaching Mathematics following problem–solving approach in secondary schools in Murang'a County developed a positive attitude and enhanced deeper understanding hence improving students' Mathematics achievement.

The problem–solving method developed a Mathematics teaching approach through enquiry process. In this procedure, able facilitators encouraged students to explore their work engagement in conceptual development. The tasks designed to involve students to accomplish intended concept. Problem – solving enables fostering of students' inquiry method, which leads to building a classroom community. These communities composed of young mathematicians who enabled to establish a society culture, which practices problem–solving approach in a collaborative atmosphere. Students' commitment to asking questions and finding out answers boosted them to discover a variety of ways to solve different tasks. Responsibility and challenges to overcome built-up this culture through interrogations in rich Mathematics classrooms rested on the shoulders of informed instructors.

5.4 Recommendations

The study has established that using problem–solving approach in public secondary schools in Murang'a County improves students' achievement on Mathematics. The method of delivering learning Mathematics improving both performance and attitude for better results was using problem – solving compared to conventional methods. The study has considered the following recommendations made according to existing practice and non – existent policy that needs formulation and enhancement.

5.4.1 Recommendations for Practice

Mathematics teachers in secondary schools should embrace using problem–solving approach to improve students Mathematics achievement. Teachers should emphasize application of problem–solving strategies to change pedagogical skills employed in teaching Mathematics. Mathematics assessment must focus more on thoughtful conceptual understanding rather than on mastery of algorithms. Students need more time to manipulate mathematical concepts to arrive at reasonable solutions. The study correctly answers the question; what public secondary schools in Murang'a County need in order to improve students Mathematics achievement. The development of a required deeper understanding of Mathematical concepts produces results. Students must actively participate in the expansion of complicated Mathematics knowledge through encouragement in search of answers.

Problem – solving does not govern students' attention to particular aspects of content, but ways of processing, communicating and disseminating mathematical information. Tasks would be challenging to invite examination of serious speculation besides hard work. There was significant Mathematics improvement, which activated students' intellectual potential context and development. The results analysis indicated that students taught using conventional methods had significantly lower achievement than those taught using problem–solving technique. Students taught using problem–solving strategy portray better mathematical conceptual understanding. This stretches student ample chance to solidify and extend what they know through classroom interactions. The collaboration in the interactive environment stimulates their learning in Mathematics.

This study focused on improving students' achievement in Mathematics through teaching Mathematics making use of problem–solving strategy. The conventional methods that emphasize on self - discovery of knowledge and interactions of students could deploy problem – solving strategies. Students who received instruction through problem – solving had acquired more self – driven information, had better connections with peers thus able to conceptualize instructions, raising their achievement scores. Therefore, the study recommends that students in secondary schools in Murang'a County taught Mathematics through problem–solving approach that enhances the students' cognitive learning domains and improves their achievement in Mathematics.

The constructivists believe that a well-planned classroom discourse holds the views that learning takes place because of number of different instructional practices. These include giving students' confidence usage of experiments in real – world problem – solving. The learners enabled to create knowledge, reflect on ideas and talk about what they have done including their understanding in transforming Mathematics knowledge application in other fields. Constructivism does not dismiss active role that a teacher plays, nonetheless, modifies his/her action as a facilitator who assist students to construct knowledge rather than reproducing already known facts. Teachers as advanced organizers provide tools necessary for problem – solving via inquiry – based learning activities. These actions help students to formulate and test ideas articulated to draw valid inferential conclusions.

Students share learnt knowledge in an environment collaboratively to advance Mathematics understanding. Constructivism transforms student from passive recipient of information to active participant in learning progress in a classroom community. This procedure promote student to retain mathematical concepts and improve Mathematics achievement. Therefore, this study recommends that facilitators should employ constructivism theory in their classroom discourse.

The study recommends that problem–solving method in teaching mathematical concepts by teachers in Murang'a County need to be encouraged towards utilization. This ought to designate the work done by creating a learning environment where students require problem understanding, devise the plan, carry out the plan and reflect on the work they have done. Murang'a County Mathematics teachers should meet regularly for trainings, workshops and seminars where they can review their work progress. These meetings would arranged to enhance their understanding and knowledge on problem–solving strategies.

Mathematics teachers use strategies that enhance students' problem – solving through collaboration of other instructional methods. The integration of Mathematical problem–solving techniques considered best method subsequently learner – centered as active participant. Deep understanding of Mathematics problems would be enhanced which lead to relational lifelong problem – solving in real world. Problem – solving through other conventional methods supports students with additional features such as cooperative learning and small groups. The students should guide to work individually, in small groups, and as a whole class to promote interaction where peer influence learning is experienced. Teachers' preferred strategies given better weight as they improved to incorporate problem– solving in secondary schools of Murang'a County.

Study recommends that teachers should provide students with opportunities to interact in a favorable rich environment to solve problems. Students should actively participate to finding individualized solutions applying problem–solving strategies. This happens when teachers encourage and give opportunities to students to share and compare their answers. Students further contrasted their methods as a general idea of this research where problem – solving involved individual solution with activities in small groups and whole class. This enhanced through classroom interaction and creating class communication providing students with confidence in problem – solving. This increases student Mathematics achievement through problem–solving activities through problems and assignments.

Problem–solving approach should viewed as a continuous examination process that always remain exposed for further modification. Learner – focused outlook that stresses learner's construction of mathematical knowledge through social interaction where knowledge is wholly developed. This improves our humanity aspect of social life where people live together. The teacher's ability to understand learners' strengths enables her/him to support the learner effectively to develop social skills.

Research findings specified that certain teaching strategies worth considerations to support problem–solving approach. According to Tripathi (2009) suggested that problem – solving in Mathematics engages a significant tool for cognitive development. This study recommends improving students' Mathematics achievements by using problem–solving practices. Student understanding and awareness of how to construct new Mathematics ideas significantly improved knowledge. The familiar intuitive solutions with student become a useful tool when planning and Management of the problems.

The development of good learning environment for learning, availability of technological devices useful for computation together improves intrinsic motivation in Mathematics. Computers and calculators when applied in Mathematics learning improve computational power. Failure to learn Mathematics hinders individual use of computational tools limiting their ability and hampering natural growth. The students in secondary schools require substantial change in Mathematics performance to join tertiary institutions and advanced courses. Mathematics proficiency demands substantial change undertaken carefully and deliberately so that children get an opportune assistance in their actions undertaken through problem – solving.

The goal of problem – solving instructional provides better way of disapprovingly think about Mathematics learning instead of applying conventional strategies. These offer learners with means to know while being enabled to engage in Mathematics independently. Students with reasonable understanding of Mathematical concepts given attention in Mathematics ideas, to clearly reason out their thoughts, be in contact with Mathematics subject as well as having flexible reasoning in various disciplines.

Finally, but not least, parents, teachers and school administration must work together to establish new higher standards in cooperation and teamwork toward the common goal of improving Mathematics achievements. This encourages students to pursue their future careers without hindrances. The parents should support teachers' assignments and homework through provision of materials required for practical application of concepts. Whenever reasonable and clearly related activities involving students', motivation perform better in Mathematics achievement as well as educational objectives.

5.4.2 Recommendations for Policy

Problem–solving approach proved effective method of Mathematics instruction and learning compared to conventional methods. Therefore, Mathematics teachers at secondary schools in Murang'a County should embrace teaching by means of problem–solving method in order improve academic achievements of students. Kenya institute of curriculum development (KICD) ought to transform current textbooks of Mathematics in conventional based learning towards problem–solving learning. Existing curriculum would improve towards achievement of teaching through problems – solving rather than conventional methods.

Mathematics educators in training institution encouraged to prepare teachers in using problem–solving learning approach. This would build student teachers confidence in applying problem–solving strategies in their future classrooms. Government through KICD should develop a curriculum with problem–solving oriented methods appropriately improve students Mathematics achievement in secondary schools. Mathematics teachers in secondary schools would enabled in employment of problem–solving method in their classrooms through extensive training programs, seminars and workshops organized for conversation growth.

Recently, the national Mathematics education reforms in Kenya focused on elements of performance in relationship to test, data and inspection. This study principally emphasis on Mathematics achievement comprising of attitude and performance. Examination on students' attitude change in Mathematics had shown relationship in performance increase. Research approved that when students bring into play available resources, provided with opportunity to practice, share their own ideas, and actively participate together in joint venture of knowledge construction they improved in performance. Study complied by Vygotsky (1978) showed that complex knowledge and skills learned through partial social interaction in construction work is a means of accomplishing reasonable lifelong education goals.

This study recommends that problem–solving strategies in social constructivism would improve Kenya's philosophy enshrined in Vision 2030. The national objectives of fostering nationalism, patriotism and promote national unity through positive contribution to life on national cohesion. Productive and operative role in coexistence in the life of Kenyans by encouraging political, social economic, technological and industrial requirements in national development.

The government has a task of training of Mathematics teachers who would use problem–solving approach to improve Mathematics achievement in secondary schools. The government must provide motivational incentive that attract and retain competent, fully prepared qualified teachers. The schools should assure Mathematics teachers of ability to retain them in good classroom environment. The favorable conditions adapted including a reasonable class size would lead to effective teaching.

The teacher-educators should put emphasis on teacher preparation using problem – solving so that the processes comprehended and practiced by teachers. The essential Mathematics content developed fully integrated in teaching developments. Learning activities should involve teachers' trainee allowing them opportunities to form connections between procedures and outcomes of problem – solving. When trainees are inspired to follow problem – solving during their course, they would build capacity and confidence in usage.

Teachers are encouraged to acquire new responsibility of teaching and self – reflection as a substitute of merely taking courses on instructional methods (Angier and Povey, 2010). Mathematics teachers taking more courses procedural teaching might improve their understanding of secondary school teaching enhancing their delivery. Mathematical ideas and connections upgraded in instructional situations when teachers necessitated with opportunities to analyze and execute problem–solving strategies (Pelczer, et.al, 2014). On daily basis discussions about teaching through problem–

solving in secondary school setting was significant since conversation on knowledge transfer from concrete to contextual concepts rather than abstract relation without context. This offered assistance teachers in solidifying and improving problem–solving strategies in their classrooms.

Mathematics curriculum sensitization on problem–solving effects on students learning Mathematics should use instead of conventional methods. Investigations of mathematical concepts growth through guided discovery and collaborations should emphasized. A community of young mathematicians should built in well-organized classrooms. Make use of available technology as well as available real materials to improve mathematical environment. The textbooks and demonstration of practical algorithm could also applied in teaching problem – solving. This could be by enhancing students' understanding and investigating Mathematics concepts.

Non – existent policy of teaching Mathematics for competency, which could improve using problem–solving approach, needs formulation. The practical interventions operating in secondary schools teaching of Mathematics in Murang'a County geared towards achieving good results. Study recommends that specific policy considerations involving Mathematics teachers put into practice implementation. Existing Mathematics text – books where students encounter exercises applying certain algorithms improved. Therefore, through policy formulation text – books would progress with appropriate problem–solving strategies.

Currently, the government policy is on competency-based education. This study highly recommends problem – solving in Mathematics to play crucial part to draw on society to appreciate the goals of education. Situational learning of making teachers strive to have activities involving students in learning process more directly ensures active participation. These activities include facilitating students to have broad perspectives of looking at different options through the help of other students' opinions. Therefore, implementers create an environment aligned to social constructivism strategies rather than traditional philosophies promoting investigation-based learning. Progressive learning process should move from traditional, teacher – centered to learner – centered.

The Model involving students is integrating different approaches into complete answer through creating new Mathematics concepts. The type of learning that takes place only by creating an atmosphere suitable for building knowledgeable learning communities should involve collaborative learning. The communities cultivate a dynamic environment through interdisciplinary learning and generating high profile activities for better learning experiences. Students appreciate these approaches since there remains clear negotiation of goals and methods of learning between students and teachers critically. The complex tasks existing in real life are comparable considering the human and environmental impacts in solution to situations. The task solutions evaluated by students concerned giving reasonable reflections on relevant meaning of what is learned. Engagement in real life task reflection into activities conducted for learning provide competency education, which is the gist of this study.

A number of questions asked by students and teachers of Mathematics achievement had obtained answers in the past in relation to low Mathematics performance. These identified as factors including teachers and students' attitudes, teachers' proficiency in pedagogical skills, content knowledge and motivation to learners and the influence of mathematical language deficiency from both students and teachers. This study recommends that all these questions could adequately be answered using problem–solving approach. This remained the unique approach in which, learners improved

Mathematics achievement in Murang'a County. The study recommends that stakeholders should hold on methods aiming at better and competent future learners' application in life skills.

5.5 Recommendation for Further Research

The study recommends further study on the following:

i. Research could be done to determine integration conventional strategies of teaching and learning Mathematics on students' Mathematics achievement.

ii. The processes and types of resources required to invoke conceptual development when problem - solving strategy used in teaching Mathematics need investigation.

iii. A study to investigate teachers' role in supporting students with problem–solving strategies in teaching and learning Mathematics.

iv. To assess long-term understanding of the mathematical concepts when students taught using problem - solving.

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APPENDICES

Appendix A: Introductory Letter

Gakinya Kigamba
P.O. BOX 00342 -1000
Thika

Principal /Teacher

Dear Sir/Madam

I am University tutor at Mount Kenya University, who would like to conduct an experimental research in form three in teaching of Mathematics. I am writing to ask you to allow me to use your school to test your students on question on the problem – solving in Mathematics. The questions are developed from KCSE syllabus to monitor students' attitudes and ability to use problem – solving in learning Mathematics. I will discuss teaching strategies for problem–solving and measuring students' attitudes with a pre and post assessment with class teacher. I would like your permission to use my instruments with your students to collect data on their attitudes and problem – solving of Mathematics achievements.

I am carrying out a study on "Effects of Problem-Solving Approach on Students' Achievement in Mathematics in Secondary School in Murang'a County, Kenya" I would be grateful if you could assist me in filling out this questionnaire. The information gathered will provide crucial knowledge that will greatly assist in the study to identify the preferred instructional strategies in teaching. However, your information treated as highly confidential and shall use for research purposes and improvement of teaching Mathematics in the County. Please do not write your name. Thank you for your time.

Yours faithfully,

Gakinya Kigamba

Email: kigambagakinya@yahoo.com

Appendix B: Consent Form for Teachers

Name of Researcher: Gakinya Kigamba John,	kigambagakinya@yahoo.com
Title of Study: Effect of Teaching Mathematics Through Problem-Solving Approach on Students' Achievement in Secondary School Level in Murang'a County, Kenya	

Kindly go through and fill this form cautiously. In case you opt to partake in this investigation, tick the suitable answers appending your signature and date the statement. In case you do not understand any item, get in touch with me.

Purpose: The purpose of this study will be to examine the effects of problem–solving approach on students' achievements in Mathematics in secondary schools of Murang'a, County Kenya.

Benefits: This research will help Mathematics teachers to improve their teaching and students' achievement. Mathematics curriculum and policy makers develop relevant content for future of our students in problem – solving. Cross the alternative, which applies, to you.

- The investigation was reasonably clarified to me in verbal/written by researcher and any inquiries concerning procedures will be offered [YES|NO]
- I am told that the study will concern the above-mentioned title [YES|NO]
- I am aware that I may pull out from the research anyhow I wish without an explanation [YES|NO]
- I am aware that every material about me will be preserved in strictest confidentiality I will not be named in any way [YES|NO]

I am freely giving agreement to partake in this investigation, and I possess a copy of this arrangement for reference.

Signature……………………………... Day/Date………………………………
For further inquiry, you can always consult the Chairman Ethics and Review Committee (ERC), P.O. Box 342 – 01000, Thika.
Gakinya Kigamba

Appendix C: Consent Form for Students

Name of Researcher: Gakinya Kigamba John, kigambagakinya@yahoo.com
Title of Study: Effect of Teaching Mathematics Through Problem-Solving Approach on Students' Achievement in Secondary School Level in Murang'a County, Kenya

Kindly go through and fill this form cautiously. In case you opt to partake in this investigation, tick suitable answers appending your signature and date the statement. In case you do not understand any item, get in touch with researcher.

Purpose: The purpose of this study will be to examine the effects of problem–solving approach on students' achievements in Mathematics in secondary schools of Murang'a, County Kenya.

Benefits: This research will help students to improve their Mathematics achievement, which as an essential subject in most career development in future.

- The examination is reasonably clarified to me in verbal/written by researcher [YES|NO]
- I am told that the study will concern the above-mentioned title [YES|NO]
- I am aware that I may pull out from the research anyhow I wish without an explanation [YES|NO]
- The test given will be accordance to the content taught in secondary school [YES|NO]
- I am aware that every material about me will be preserved in strictest confidentiality I will not be named in any way [YES|NO]
- Am aware that am free to withdraw consent and to discontinue participation in the study at any time without prejudice to other participants, [YES|NO]

I am freely giving agreement to partake in this examination, and I possess a copy of this arrangement for reference.

Signature…………………………………. Day/Date……………………………………

For further inquiry, you can always consult the Chairman Ethics and Review Committee (ERC), P.O. Box 342 – 01000, Thika.

Gakinya Kigamba

Appendix D: Pre-test Achievement Test

This questionnaire has 20 items. Answer all questions showing all necessary working, marks awarded in clear and organized working.

1. Using the rearranging principle to do calculations without using calculating devices.

i.evaluate -8 – (-14) + 6 + (-22)

ii.simplify $\frac{-32 \div 4 - (8 + -6)}{3 + (-2 \times 5) - (2)}$

iii.if a = $\frac{b(100 + c)}{100 - c}$, find b when a = 9 and c = 20

iv.Complete the list, describe the pattern and write a general formula of the sequence:

Natural numbers	Number of terms	Natural numbers	Sum
5	1	5	2
5, 8	2	5 + 8	13
5, 8, 11	3	5 + 8 + 11	24
5, 8, 11, …10^{th} term	10	5 + 8 + 11 + …10terms	

Column (1) has a common difference, d find it. Multiply the difference with column 2 to obtain the products. What is the relationship between the products and numbers in column (1)? Reverse the order of numbers in column (3) and add it together what is the relation of the sum in column (4) and new sum. Use your relationships to establish a formula to generalise third and fourth column.

I.5, 8, 11, …, nth term

II.5 + 8 + 11 + … + n

2. Algebra

i.Simplify these expressions 4(8d -3) + 5 (3 -2d)

ii.If a = 5, b = 3, c =1 and d = 0, find the value of $\frac{a+c}{b-cd}$

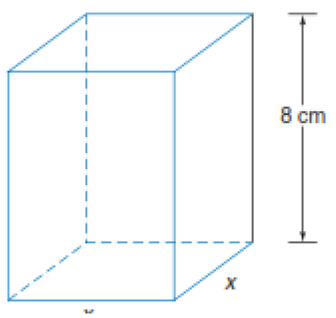
iii. Solve 5x + 4 = 34

iv.Solve 1/3x - 2 = 4 - ½x

3. Adding “tens” and ones separately find the sum of 37 and 26

4. What meant by an edge, perimeter, a square unit and the area?

5. The total surface area of the right square prism shown below is $210cm^2$. Find the length of the side base if the attitude of the prism is 8cm.



Cuboid A

6. Simplify the expression $\sqrt{(10^2 - 5^2)}$

7. Work out following.

i. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

ii. $2\frac{1}{2}+\frac{7}{10}-\frac{2}{5}$

iii. $8\frac{3}{4}\times1\frac{3}{5}\div4\frac{2}{3}$

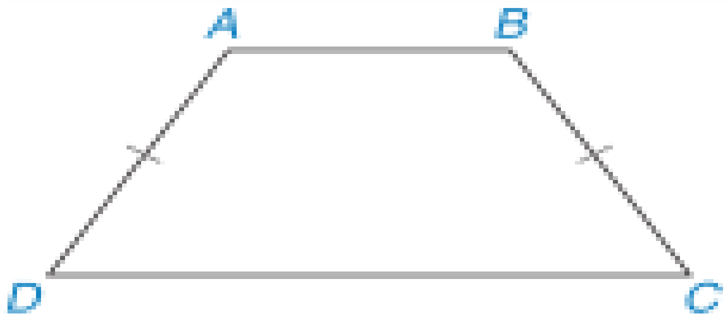
8. Using the rearranging principle to do calculations without using calculating devices.

i. 276 + 84

ii. 7 x (9 + 4)

iii. 5.14 + -3.709 + 13.3

9. A woman buys 5kg of potatoes at p shillings per kg and 2 cauliflowers at q shillings each. Find change from Ksh 500, which she should receive.



Trapezium A

10. Given isosceles trapezoid ABCD with AB parallel to DC. Find the measures of the angles ABCD if angles A and B are 12x + 30 and 10x + 46 respectively.

APPENDIX E: POST – TEST ACHIEVEMENT TEST

This questionnaire has 20 items. Answer all questions showing all necessary working, marks awarded in clear and organized working.

1. Using the rearranging principle to do calculations without using a calculating devises.

i.evaluate -7 – (-22) + -5 - (-8)

ii.simplify $\frac{-42 \div 3-(-6+-8)}{9+(-7\times 5)-(-12)}$

iii.if a = $\frac{b(100+c)}{100-c}$, find c when a = 6 and b = 6

iv.work out the cubes from 1^3 to10^3

Copy and complete this pattern

Natural numbers	Sum	Cubes of natural numbers	Sum
1	1	1^3	1
1 +2	3	$1^3 + 2^3$	9
1 +2 +3	6	$1^3 + 2^3 + 3^3$	36
…	…	…	…
1 + 2 + 3 …+ 10	55	$1^3 + 2^3 + 3^3 + \ldots + 10^3$	

What do you notice about the connection between the 2nd and 4th columns? Double the numbers in column (2) and divided each by the largest number of the same row in column (1). What do you notice? Can you use this to find a formula for?

i. 1 + 2 + 3 + … + n

ii. $1^3 + 2^3 + 3^3 + \ldots n^3$

2. Algebra

i.Simplify these expression 2(2a -3) - 3 (2a +b -4) + 4(a + 2b) ii.If a =5, b= 3, c =1 and d = 0, find the value of $\frac{a^2 - b^2 + d^2}{(b+c)^2}$

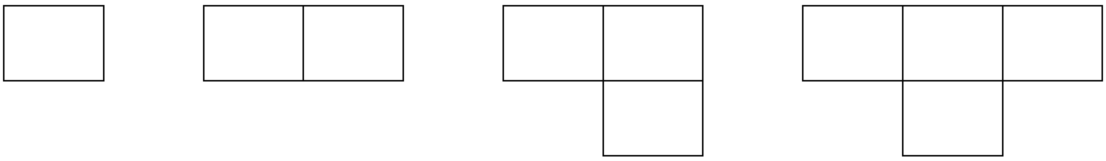
Solve these equations

iii.3h = 90 - 2h

iv.3(2p -1) = 39 – 2(p + 1)

3. Adding “tens” and |ones” to find the sum of 48 and 25, using at least seven different solutions methods

4. Add one square tile at time along one edge. Find perimeter according to number of match sticks used to confirm the numbers. Describe the pattern and find the general formula.



P = 4 p = 6 p = 8 p = 10

5. Simplify the expression $\sqrt{(39^2 - 36^2)}$

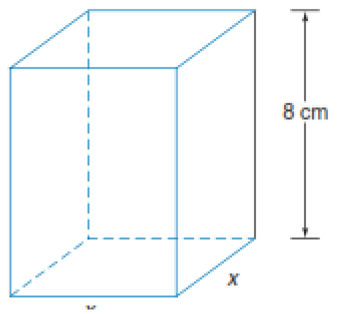
6. Work out following

i. $\frac{5}{8} + \frac{1}{6} + \frac{1}{4}$

ii. $1\frac{3}{4} - \frac{4}{5} + 2\frac{7}{8}$

iii. $(3\frac{1}{7} \times 8\frac{3}{4}) - 2\frac{1}{3}$

7. The total surface area of the right square prism shown below is $1554cm^2$. Find the length of the side base if the attitude of the prism is 8cm.



Cuboid B

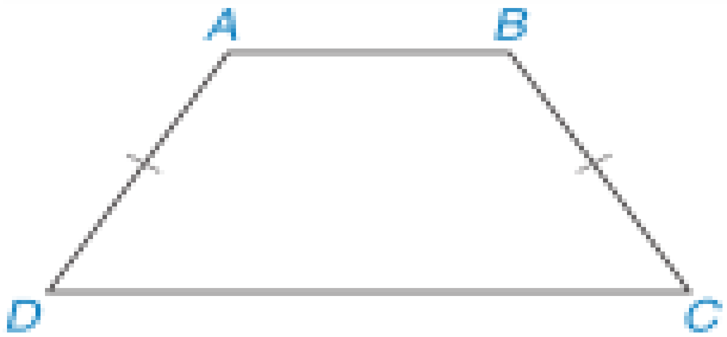
8. Using the rearranging principle to do calculations quickly without using calculating devices.

i. 76 + 83

ii. 8 x (9+5)

iii. 15.34 + 12.67

9. 20 people went together to an exhibition. There were x adults, for whom the entrance fee was Ksh 300, and rest were students who charged Ksh 200 each. The total cost was Ksh 52000 form an equation for finding x.



Trapezium B

10. Given isosceles trapezoid ABCD with AB parallel to DC. Find the length of each diagonal (not shown) if it is known that AC = 2y – 5 and BD = 19 – y

Appendix F: Questionnaire for Students

This instrument consists of number of statements, which are behavior and confidence of students when using intervention of problem – solving in Mathematics in secondary schools in Murang’a County, Kenya. Using the answer key shown below, you asked to respond to each statement: please read each question carefully and choose only ONE response. You may ask help if you do not understand something or if you are not sure how to respond. There is no right or wrong answer to any of these items. The only correct responses are those that are true for you. Please give you frank opinion. Your responses treated confidentially and used for research purpose only.

<u>Example</u>

		Totally disagree	Partially disagree	Uncertain	Partially agree	Totally agree
I use exercises given for homework to prepare for the introduction of a new Mathematics concept		1	2	3	4	5

In making, your judgments consider the answer you agree with most. Please respond to every method by circling one number.

	Totally agree	Partially agree	Uncertain	Partially disagree	Totally disagree
1. I focus on Mathematics class	5	4	3	2	1
2. I would be happy to be recognized as an excellent student in Mathematics	5	4	3	2	1
3. I make an effort to respond to Mathematics problem the teacher asks	5	4	3	2	1
4. I make mistakes in a Mathematics problem; I work until I find the correct solution.	5	4	3	2	1
5. If I cannot do a Mathematics problem, I keep trying different strategies.	5	4	3	2	1
6. If I can't do a Mathematics problem, I seek help to solve the problem	5	4	3	2	1
7. I have a mind built for Mathematics	5	4	3	2	1
8. I can get good grades in Mathematics.	5	4	3	2	1
9. I know how to do with difficult problems in Mathematics.	5	4	3	2	1
10. I know how to solve Mathematics problems	5	4	3	2	1
11. I know strategies that can help me solve Mathematics problems	5	4	3	2	1
12. I am confident with my Mathematics skills and strategies	5	4	3	2	1
13. I am interested to become skilled at new strategies in Mathematics	5	4	3	2	1
14. I get reward for my efforts in Mathematics.	5	4	3	2	1
15. Learning Mathematics is fun for me.	5	4	3	2	1
16. I feel accomplished when I solve Mathematic problem correctly.	5	4	3	2	1
17. I can get good grades in Mathematics	5	4	3	2	1
18. I have a lot of self –confidence when it comes to Mathematics	5	4	3	2	1
19. I am good in Mathematics	5	4	3	2	1
20. I feel confident when I am explaining Mathematics concept to others in my group.	5	4	3	2	1

Appendix G: Questionnaire for Teachers

Please, you requested complete honestly, response to this questionnaire to the following questions to the best of your knowledge according to the instructions given and they consumed for research only while kept confidential.

1. Please, indicate the number of candidates who obtained the following grades in their 2015, 2016, and 2017at KCSE in Mathematics National results of your school

GRADES	2015	2016	2017
A - A			
B – B+			
C+ - B-			
D+ - C			
E - D			
TOTAL			

Please mark one answer for each of the information questions below that apply to you.

2. Indicate the one that best describes your professional qualification.

(a) Diploma

(b) B.A. or B.Sc. graduate

(c) B.A. or B.Sc. with education (PGDE)

(d) B.Ed.

(e) Others (specify)___________________________

3. Indicate the number of years you have taught in secondary schools

(a) Less than 1 year

(b) 1 to 3 years

(c) 4 to 6 years

(d) Over 6 years

4. Indicate the class or classes you usually teach in the present school.

(a) Form one

(b) Form two

(c) Form three

(d) Form four

This instrument consists of 24 statements of possible methods of teaching and learning students to perform well in Mathematics at secondary schools in Kenya. Using the answer key, you asked to respond to each statement in two different ways:

First: - How often you use the strategy in your Mathematics lessons.

Then; - In your judgment, how often the strategy should be used in Mathematics lessons.

Example

		Not followed	Followed only infrequently	Followed often	Followed very often	Followed all the time
To use exercises given for homework to prepare for the introduction of a new Mathematics concept should be	Is	1	2	3	4	5
	Should be	1	2	3	4	5

In making, your judgments consider the instruction of mathematics as a whole. Please respond to every method by circling one number after is and another should be methods of teaching.

	Methods of teaching		Followed all the time	Followed very often	Followed often	Followed only infrequently	Not followed
1.	To explain and demonstrate a mathematical concept by use of an example	Is	5	4	3	2	1
		Should be	5	4	3	2	1
2.	To collect and correct set homework after school in the staffroom or elsewhere	Is	5	4	3	2	1
		Should be	5	4	3	2	1
3.	To provoke students' thinking and responses by asking probing relevant questions that lead to a predetermined concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
4.	To facilitate in a discussion among students in an attempt to arrive at the answer to a mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
5.	To ask leading questions with an aim of helping students to establish a mathematical procedure	Is	5	4	3	2	1
		Should be	5	4	3	2	1
6.	To respond to answers and questions from students and finally summarizing the points to arrive at the intended mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
7.	To illustrate how a mathematical example(s) is/are worked out on the blackboard by the student(s) while the rest watch and questions	Is	5	4	3	2	1
		Should be	5	4	3	2	1
8.	To get students solve their mathematical problem(s) by investigating its/their solution(s)	Is	5	4	3	2	1
		Should be	5	4	3	2	1
9.	To let students, construct handle concrete materials as they learn a new mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
10.	To introduce a mathematical concept, the students are given problems to work out as the teacher circulates in the classroom giving necessary help as students work	Is	5	4	3	2	1
		Should be	5	4	3	2	1
11.	To let students, construct models which they will use in development of mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1

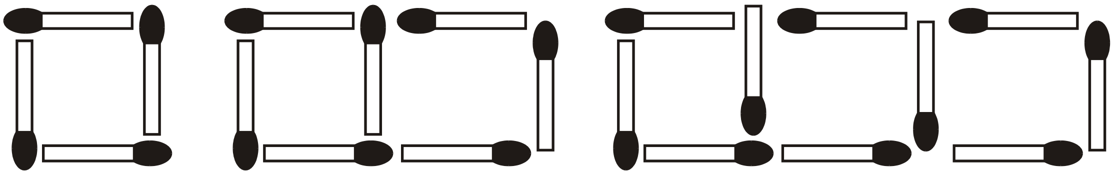
12.	To show how a mathematical concept(s) is /are developed by working out a related example(s) on the blackboard by the teacher while students watch and ask questions	Is	5	4	3	2	1
		Should be	5	4	3	2	1
13.	To use discussion after mathematical programs shown on the television or video	Is	5	4	3	2	1
		Should be	5	4	3	2	1
14.	To apply mathematical knowledge learned in class to unfamiliar situations	Is	5	4	3	2	1
		Should be	5	4	3	2	1
15.	To listen to tape recorder or radio Mathematics lesson is a useful way of introducing a mathematical structure	Is	5	4	3	2	1
		Should be	5	4	3	2	1
16.	To give students a real-life problem and let them investigate its solution	Is	5	4	3	2	1
		Should be	5	4	3	2	1
17.	To show students concrete object(s) in order to help them see the relationship between mathematical concept(s)	Is	5	4	3	2	1
		Should be	5	4	3	2	1
18.	To give problems to be done as the teacher moves round the class giving feedback is a useful way of introducing a new mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
19.	To subject the students to a game such that they will discover a related mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
20.	To start with real life problem and proceed to answer it in introducing a new mathematical concept	Is	5	4	3	2	1
		Should be	5	4	3	2	1
21	To enable the students to develop competence to solve problems independently	is	5	4	3	2	1
		Should be	5	4	3	2	1
22	To help the students develop positive attitude towards learning of Mathematics	is	5	4	3	2	1
		Should be	5	4	3	2	1
23	To encourage students be open, honest and trusting in their relationship with others	is	5	4	3	2	1
		Should be	5	4	3	2	1
24	To help learners to be relevant in the current society where Mathematics is a key	is	5	4	3	2	1
		Should be	5	4	3	2	1

Thank you.

Appendix H: Intervention Problems

Problem 1

Here are some patterns made from matchsticks:



Pattern number 1   Pattern number 2   Pattern number 3

Draw Pattern number 4.

(b) Complete this table for the pattern sequence

Pattern number	1	2	3	4	5
Number of matchsticks used	4	7	10		

(c) How many matchsticks used in pattern number 15?

John says that if there are *m* matchsticks in pattern number *n* then the formula for *m* in terms of *n* is $m = 4n$.

(d) Explain why John's formula is not correct.

Mr. Peter owns minibuses and coaches.

Each minibus has 12 seats.

(e) Write an expression, in terms of *m*, for the number of seats in *m* minibuses.

Each coach has 48 seats.

(f) Write an expression, in terms of *m* and *c*, for the number of seats in *m* minibuses and *c* coaches.

Problem 2

Here is a table for a two-stage number machine.
It divides by 2 then adds 3.
Complete the missing numbers in the table:

Input → **÷ 2** → **+ 3** → **Output**

Input	**Output**
2	4
4	5
6	………..
12	………..
………….	13

Problem 3

Patterns and relationships

Write natural numbers 1 - 100

Skip count in 2s and see what number pattern.

How do you guess and check the next number in a pattern?

Skip count in 3s, 5s, 10s etc.

Continue a sequential pattern.

Systematically count to establish rules for sequential patterns.

Describe skip –counting patterns.

Use graphs to illustrate skip – counting patterns.

Investigate and recognize the results of adding and subtracting combinations of odd and even numbers.

State generalisations about the addition and subtraction of combinations odd and even

Apply generalisations about odd and even number to problem–solving situations.

Describe patterns

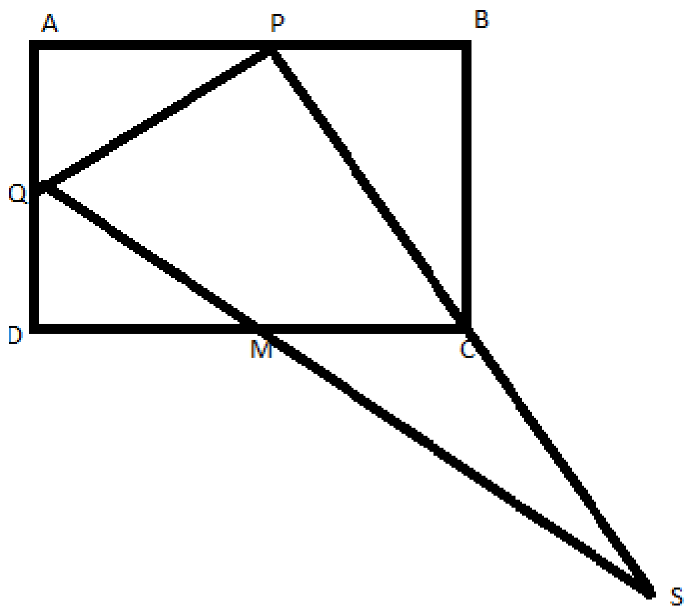
Continue a pattern

Create patterns

Record patterns on grid paper

Geometry: Problem 4

Given the square ABCD where P, Q and M are the mid –points of AB, AD and CD. If AB = 1m, find the area of triangle PQS.

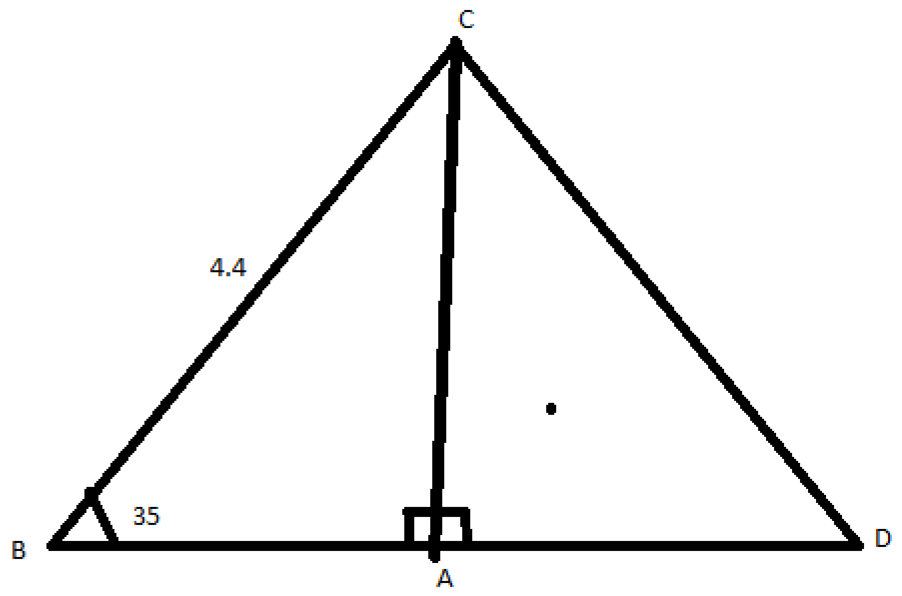


Problem 5: Twelve couples invited to a party. The couples will seat at a series of small square tables, placed end to end to form one large long table. How many of these small

tables needed to seat all 24 people? If each small table is, 2 square metres find how many squares are covered by all 24 tables?

Problem 6: Kamau is visiting his father's farm. There are goats and chickens in the barnyard. He counts 20 heads and 32 legs. How many chickens and goats are there?

Problem 7: find all the sides of the triangle using sine, cosine and tangent ratios



Appendix I: Certificate of Ethical Clearance

Mount Kenya University

JUNE 25, 2018

Ref. No. MKU/ERC/0893

CERTIFICATE OF ETHICAL CLEARANCE

This is to certify that the proposal titled **"EFFECT OF TEACHING MATHEMATICS THROUGH PROBLEM SOLVING APPROACH ON STUDENTS' ACHIEVEMENT IN SECONDARY SCHOOL LEVEL IN MURANG'A COUNTY, KENYA,"** Whose Principal Investigator is Mr John Kigamba Gakinya (PhDED/2014/58006) has been reviewed by Mount Kenya University Ethics Review Committee (ERC), and found to adequately address all ethical concerns.

Mr Francis W. Makokha
Secretary, Mount Kenya University ERC

Sign: ________ Date: 25·06·2018

Prof. Francis W. Muregi
for Chairman, Mount Kenya University ERC

Sign: ________ Date: 25/6/2018

The Chairman
Mount Kenya University
Ethics Review Committee
P. O. Box 342 - 0100, Thika

Main Campus, General Kago Road, P.O. Box 342-01000 Thika. Tel: +254 67 2820 000,
Cell: +254 720 790 796, 0709 153 000
Email:info@mku.ac.ke, Web: www.mku.ac.ke
Chartered and ISO 9001 : 2008 Certified Institution.
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Appendix J: Introduction Letter School of Postgraduate

Mount Kenya University

SCHOOL OF POSTGRADUATE STUDIES

PhDED/2014/58006

02nd July, 2018

The Director, Research Coordination Division
National Commission for Science, Technology & Innovation
Utalii House, 8th & 9th Floor
P.O Box 30623- 00100
NAIROBI

Dear Sir/Madam,

RE: JOHN KIGAMBA GAKINYA - REGISTRATION NO. PhDED/2014/58006

The purpose of this letter is to introduce the above named student who is pursuing **Doctor of Philosophy in Education (Mathematics Education)** in the Department of **Education Psychology and Technology** in the School of **Education.**

The title of his research is ***"Effect of Teaching Mathematics through Problem Solving Approach on Students' Achievement in Secondary School level in Murang'a County."***

He has been cleared by the University's Ethics Review Committee (Certificate attached) and now has to proceed to the field to collect data for his research between July and **December 2018.**

Any assistance accorded to him will be highly appreciated.

Thank you.

Mount Kenya University
Dean, School of Postgraduate Studies
P. O. Box 342 - 01000
Thika

Dr. Samuel Karenga, PhD
Dean, School of Postgraduate Studies
Enc.

Main Campus, General Kago Road, P.O. Box 342-01000 Thika. Tel: +254 67 2820 000,
Cell: +254 720 790 796, 0709 153 000
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Appendix K: NACOSTI Research Authorization Letter

NATIONAL COMMISSION FOR SCIENCE, TECHNOLOGY AND INNOVATION

Telephone:+254-20-2213471,
2241349,3310571,2219420
Fax:+254-20-318245,318249
Email: dg@nacosti.go.ke
Website : www.nacosti.go.ke
When replying please quote

NACOSTI, Upper Kabete
Off Waiyaki Way
P.O. Box 30623-00100
NAIROBI-KENYA

Ref: No. **NACOSTI/P/18/87433/24426**

Date: **7th August, 2018**

Gakinya Kigamba
Mount Kenya University
P.O. Box 342-01000
THIKA.

RE: RESEARCH AUTHORIZATION

Following your application for authority to carry out research on ***"Effect of teaching mathematics through problem-solving approach on students achievement in secondary school level in Murang'a County, Kenya"*** I am pleased to inform you that you have been authorized to undertake research in **Murang'a County** for the period ending **6th August, 2019.**

You are advised to report to **the County Commissioner and the County Director of Education, Murang'a County** before embarking on the research project.

Kindly note that, as an applicant who has been licensed under the Science, Technology and Innovation Act, 2013 to conduct research in Kenya, you shall deposit **a copy** of the final research report to the Commission within **one year** of completion. The soft copy of the same should be submitted through the Online Research Information System.

DR. STEPHEN K. KIBIRU, PhD.
FOR: DIRECTOR-GENERAL/CEO

Copy to:

The County Commissioner
Murang'a County.

The County Director of Education
Murang'a County

Appendix L: Research Permit

THIS IS TO CERTIFY THAT:
MR. GAKINYA KIGAMBA
of MOUNT KENYA UNIVERSITY, 0-1000 THIKA,has been permitted to conduct research in *Muranga County*

on the topic: ***EFFECT OF TEACHING MATHEMATICS THROUGH PROBLEM-SOLVING APPROACH ON STUDENTS ACHIEVEMENT IN SECONDARY SCHOOL LEVEL IN MURANGA COUNTY, KENYA***

for the period ending:
6th August,2019

Permit No : NACOSTI/P/18/87433/24426
Date Of Issue : 7th August,2018
Fee Recieved :Ksh 2000

....
Applicant's Signature

....
Director General
National Commission for Science, Technology & Innovation

CONDITIONS

1. The License is valid for the proposed research, research site specified period.
2. Both the Licence and any rights thereunder are non-transferable.
3. Upon request of the Commission, the Licensee shall submit a progress report.
4. The Licensee shall report to the County Director of Education and County Governor in the area of research before commencement of the research.
5. Excavation, filming and collection of specimens are subject to further permissions from relevant Government agencies.
6. This Licence does not give authority to transfer research materials.
7. The Licensee shall submit two (2) hard copies and upload a soft copy of their final report.
8. The Commission reserves the right to modify the conditions of this Licence including its cancellation without prior notice.

REPUBLIC OF KENYA

NACOSTI

National Commission for Science, Technology and Innovation

RESEARCH CLEARANCE PERMIT

Serial No.A 19915

CONDITIONS: see back page

Appendix M: Map for Murang'a County

EFFECTS OF TEACHING THROUGH PROBLEM - SOLVING APPROACH ON STUDENTS' MATHEMATICS ACHIEVEMENT IN SECONDARY SCHOOLS OF MURANG'A COUNTY, KENYA

by Gakinya Kigamba John

Submission date: 15-Jun-2020 11:42AM (UTC+0300)
Submission ID: 1344132497
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Word count: 50550
Character count: 296139

EFFECTS OF TEACHING THROUGH PROBLEM - SOLVING APPROACH ON STUDENTS' MATHEMATICS ACHIEVEMENT IN SECONDARY SCHOOLS OF MURANG'A COUNTY, KENYA

ORIGINALITY REPORT

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