Construction of Markov Transition Matrices for Cohorts of Students in Bsc.Actuarial Science Programme

(A Case Study of JKUAT)

Imboga O. Herbert, George O. Orwa and H.M. Humpreys
Jomo Kenyatta University of Agriculture and Technology
Department of Statistics and Actuarial Science
P.O Box 62000-00200, Nairobi, Kenya
Email:imbogaherbert@yahoo.com

Abstract

This paper presents an application of Markov Analysis of student flow in a higher educational institution. In the education system not all students who begin an academic year stay till the end. Many of them invariably drop out for a variety of reasons. A transition model is one which describes the stocks and flows of students through an education system in terms of transition ratios. In this paper we describe one such model which traces the flow of a cohort of students through the system. In this study, the model considered is that of first-order Markov chain. Also the particular Markov chain studied here has a finite number of states and a finite number of points at which the observations are made. It is shown that under fairly general Markov chain model of the transitional determination, student flows do not display the random walk characteristics which may be interpreted as purely following a Markov process.

***Key words*:** Fundamental matrix, Markov analysis, recurrent state, Transition probabilities, Transient state.

1.0 Introduction

Education can be considered as a hierarchical organization in which a student stays in a given school stage for one academic year and then moves to the next stage or leaves the system as a graduate or drop out. In the study of a cohort, a transition model is used as an application of the more general Markov chain model in studying the flow of groups of students through an academic education system. The term cohort will be used to denote a group of students regardless of age or socio-economic background who enter first year in the same academic year. The students graduate from the system, transit from one grade year to the next higher grade, repeat the same grade or drop out of the system due to factors like lack of fees, sickness and poor academic performance. Thus students finally enter permanent states as graduates or drop outs. The student can either graduate at the final stage or drop out at any of its stages. The particular characteristics of interest are:-

i. The transition probability matrix
ii. The estimates of the probabilities that a student will graduate from a system after spending (1,2,3,4) years of study respectively.
iii. The expected length of schooling.

My interpretation, to be named Herbert Interpretation of Completion Rates (HICOR), allows for students joining a four year programme in year n, who eventually complete their programme successfully in year $n+\ 4, n+5\ or\ n+6$, i.e. some may complete after maybe repeating a year of study or taking academic leave for one year. This differs from what I would name

Interpretation of Completion Rate (JICOR), in which the only successful completions are those of students, who joined in year n and completed their programme successfully in $year\ n + 4$..

1.1 Context of the Problem

The official policy outlining the program of study at Faculty of Science requires students that attend continuously, realistically; however many students do not attend for 8 consecutive semesters. They may lack the intellectual or academic desire necessary to study. They may be academically dismissed for one semester until they are reinstated to the university system for one more chance. Because of these and other social and economic factors, it was decided that a study of student flow at the Faculty of Science was needed so that some objective basis would exist to understand and explain the nature of student progression, to evaluate the options available to admission policy and decision maker's and recommend appropriate adjustment to existing policies. The research paper is designed to answer the following questions:

i. At any given time, what is the probability that a student who has been admitted will graduate?
ii. At any given time, what is the probability that a student who has been admitted will withdraw from course at any one given stage?
iii. What is the average length of time a student will spend in the program at each stage level?

1.2 Assumptions

i. New enrolment in the education system is only through the first year. For students joining the University as mid entry students in second or third year it will be assumed that they passed all units of the previous year(s) at the first attempt.
ii. A student cannot jump to a higher grade or be demoted to a lower grade of the system, so that $Pij(t) = 0$ for all $j = i + 2, i + 3$ and for all $j < i$
iii. A student cannot repeat a given year more than once. However a student may be readmitted any number of times after having difficulties of say fees or illness however the probability of infinite readmission is very small.

From the model it's clear that at the end of each academic year, some of the students pass and thus move to the next higher grade the following year, with probabilities $Pi, i + 1, i > 1$ or fail to pass and thus repeat the same grade with probabilities $Pi, i\ i = 1; 2; \ldots$

The transition probability satisfy

$p_{i,i} + p_{i,i+1} \leq 1, 0 \leq p_{ij} \leq 1, i \leq j$

Let $ni(t)$ be the number of students belonging to a given cohort, who are enrolled in school grade i in year t, and $nij(t + 1)$ be the number of students who move to the next school grade j the following year. Then by the maximum likelihood principle, the transition probability Pij may be estimated from

$$p_{ij}(t+1) = \frac{n_{ij}(t+1)}{n_i(t)} \quad , j = i + 1; \ldots; S$$

Where S is the maximum number of years that the student will spend in the program of study.

2.0 Literature Review

A Markov chain is a sequence or chain of discrete states in time or space with fixed probabilities for the transition from one state to a given state in the next step in the chain. In the simplest form, a Markov chain may be regarded as a series of transition between different states, such that the Probabilities associated with each transition depend only on the immediately preceding state, and not on how the process arrived at that state. Such a chain contains a finite number of states, and the probabilities associated with the transition between the states do not change with time, i.e. they are stationary.

A first-order Markov chain takes account of only a single chain in the process, but the definition may be extended so that the probabilities associated with each transition depend on the events earlier than the immediately preceding one. Furthermore, the Markov chain may exhibit multiple dependence relationships, so that the probabilities associated with each transition depend jointly on more than one previous event.

The problem of understanding and assessing the flows of the students through educational systems has long been dealt with using finite absorbing Markov chains. Early work by Burke (1972) and Nicholls (1983) analyzed the flow of student teachers at the undergraduate level and the flow of students at undergraduate and graduate level within a business faculty.

Subsequently there appears to have been little work undertaken in this specific area with the exception of shah and Burke, (1999) who published an absorbing Markov chain analysis of undergraduate students in the Australian higher education system. At the graduate level, Nicholls (1983) and (2007) has investigated two types of graduate students systems using Markov chains both in Australia. It is worth noting that, as Nicholls (2007) points out, large numbers of application of absorbing Markov chains in allied areas, for example manpower planning models in academics (Hopkins and Massey, 1981) academic planning (Bowan and Schuster, 1985), assessing personnel practices in higher education (Baker and Williams, 1986), faculty structure (Ling and Pen – Gouzhung, 1987) and again, manpower planning (Hackette *et al.* 1999) have been undertaken.

3.0 Background of the Markov Chain Theory

Markov chain theory is one of the mathematical tools used to investigate dynamic behaviors of a system (e.g. workforce system, financial system, health service system) in a special type of discrete time stochastic process in which time evolution of the system is described by a set of random variables. It is worth mentioning that variables are called random if their values cannot be predicted with certainty and discrete time means that the state of the system can be viewed only at discrete instants rather than at any time. The type of discrete time process applicable by Markov chain theory is called the Markov process which is defined as

$p(X_{t+1} = x_{t+1}/\ X_0 = x_0, X_1 = x_1, X_2 = x_2 \dots X_n = x_n)$ ....(1)

Where $\{X_i = x_i \ \ i = 0.1.2, \dots \dots \dots\}$

means that the random variable Xi (uppercase) has the value xi (lower case) at the time i, and P is the conditional probability distribution of the system.

Equation (1) simply states that the conditional probability of the system in the state $t + 1$ at the moment $t + 1$, given by the left hand side of equation (1), is independent of the states occupied before the moment. In other words, the probability of which state a Markov process will be in at the next moment is determined by the current state only and is independent of evolution history of the system. The right hand side of Equation (1), $p(X_{t+1} = x_{t+1}/\ X_t = x_t)$, is called a one-step

transition probability which describes the chance a system can transit to state X_{t+1} at the moment $t + 1$ given xt, the status of the system now at time t.
Denoting $Pij(t)$ as the transition probability of a system from state i to state j, the transition matrix is defined as

$$P(t) = \begin{pmatrix} P_{11}(t) & P_{12}(t) & \dots & P_{1k}(t) \\ P_{21}(t) & P_{22}(t) & \dots & P_{2k}(t) \\ \vdots & \vdots & \dots & \vdots \\ P_{k1}(t) & P_{k2}(t) & \dots & P_{kk}(t) \end{pmatrix} \text{..............................}(2)$$

Where k is the number of exclusive states of the system
The matrix is P (t) is called stochastic for

$$0 \le Pij(t) \le 1$$

And $\sum_{j=1}^{k} p_{ij}, (i, j = 1,2 \dots\dots k)$

We note that some authors use the name "Markov chain" for those with time independent (or stationary) transition probability. The chains with transition probability varying with time are named non-homogeneous Markov chains. We will assume that the education system is time homogenous so that there is a fixed probability Pij that a student in year t will transfer to grade j in year $(t + 1)$. This gives rise to the transition matrix

$$P = Pij, (i, j = 1, 2 \dots)$$

3.1 Development of model

The Markov model is based on an underlying stochastic process in which a system in one state, si, moves to a subsequent state, sj. The states are commonly referred to as the current state and the next state. The act of moving from one state to the next is referred to as a step or transition. It is possible to construct a transition matrix from one year of study to the following year of study based on the sequence fij , for the number of who had been in state j at the end of one year of study, and were in state i at the end of the year of study. On dividing fij by the sum of fij for a particular state j, the corresponding probability Pij that a student who had been in state j at the end of one year of study, would be in state i at the end of the following year of study would be determined.

3.2 Transition Matrices

The matrix of transition probabilities provides a compact and unique description of the behavior of a Markov chain. Each element in the matrix represents the probability of the transition from a particular state (represented by the row of the matrix) to the next state (representing the columns of the matrix). Assuming a fixed number of possible states, the transition to and from every state can be described by a single matrix. A Markov transition matrix with five states is given below, where each Pij represents the probability of the transition from state Sj to state Si . Because all events from any one state must either remain in the same state or move to one of the others, the sum of the probabilities in each column is exactly 1.0. There is, however, no necessity for the sum of the probabilities in the rows to equal any fixed value.

3.3 Initial Transition Matrix

Let the state of the system be denoted by integers $1,2,\dots N$ at times $t = 0;\ 1;\ 2..$. Let Pij denote the probability that a student in grade i at time $(t-1)$ will be in grade j at time t giving rise to transition matrix $P = ((Pij))$; $i,j = 1;\ 2\dots N$. Let $nij(t)$ represent the number of students in grade i at time $(t-1)$ who will be in grade j at time t , also, let $ni(t-1)$ represent the number of students in grade i at time $(t-1)$, then assuming the multinomial distribution, the transition probabilities are estimated from $Pij = nij\ (t)/\ ni(t-1)$
Where $i,j = 1,2,\dots N$
This is the proportion of students who were in grade i at time $(t-1)$ who end up being in grade j at time t.

4.0 Construction of Markov Transition Matrices between Two Consecutive Years of Study.
For the Faculty of Science in JKUAT, the main point of interest was to examine whether students, who at the end of a year of study were in one of five states, i.e. clear pass, pass with one failed unit, pass after sitting supplementary examinations, pass after repeating the year of study, and discontinuation/deregistration, at the end of a year of study, called a stage in a Markov transition, would continue in the same state or go to a different state after sitting their examinations at the end of the following year of study. For each cohort, it was possible to construct a transition matrix from one year of study to the following year of study, based on the frequencies fij , for the number of students who had been in state j at the end of one year of study, and who were in state i at the end of the following year of study. On dividing fij by the sum of the fij for a particular state$j$, the corresponding probability Pij that a student, who had been in state j at the end of one year of study, would be in state i at the end of the following year of study would be determined. Initially, the transition matrices were calculated for each cohort of each of the two programmes that were of interest in the study. Thereafter, a statistical test was conducted to determine whether there was evidence that the transition matrices for a particular programme of study provided evidence that the observed probabilities in each matrix could have come from a common population. In both programmes it was found that the observed probabilities in each matrix did come from a common population, and it is now proposed in a future study to test these transition matrices on a different cohort of students for the programmes.

4.1 Students Admitted to B Sc Actuarial Science in JKUAT May 2005
(Progress from First to Second Year of Study)
The frequencies $f\ i\ j$, for the number of students, in this cohort, who had been in state i at the end of the first year of study, and who were in state j at the end of the second year of study, is shown in table 1.
From the Table 1, it can be seen that, of the 68 students who passed all their units in year 1, 58 also passed all units in year 2, 5 passed with one failed unit in year 2, 4 passed after taking supplementary examinations in year 2, and 1 student was discontinued or deregistered in year 2. Of the 4 students who passed after taking supplementary examinations in year 1, 2 also passed after taking supplementary examination in year 2, 1 passed with one failed unit in year 2, and 1 had to repeat year 2, but passed during the repeat year.
The same data are now displayed as a 5X5 matrix.

$$\begin{bmatrix} 58 & 5 & 0 & 0 & 0 \\ 5 & 2 & 1 & 0 & 0 \\ 4 & 3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 4 \end{bmatrix}$$

From these frequencies, the transition matrix of probabilities can now be obtained by dividing each frequency f_{ij} by the sum of the f_{ij} for a particular state j, to give the resulting transition probability p_{ij}.

TABLE

GIVENY1 > ACTUAL Y2]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.853	0.455	0	0	0
PASS 1F	0.074	0.182	0.250	0	0
PASS SUP	0.059	0.273	0.500	0	0
PASS RP	0	0	0.250	0	0
DIS/DR	0.015	0.091	0	0	1
TOTAL	1	1	1	0	1

MATRIX

$$\begin{bmatrix} 0.853 & 0.455 & 0 & 0 & 0 \\ 0.074 & 0.182 & 0.250 & 0 & 0 \\ 0.059 & 0.273 & 0.500 & 0 & 0 \\ 0 & 0 & 0.250 & 0 & 0 \\ 0.015 & 0.091 & 0 & 0 & 1 \end{bmatrix}$$

Pre-multiplication by this transition matrix, on the probability vector for a student being in one of the five states achieved after the first year of study, will yield the corresponding probability vector for a student being in one of these five states after the second year of study. In the first year of study, the probability of passing all units was 0.782, of passing with one failed unit

0.126, of passing after taking supplementary examinations 0.046, of passing after repeating 0, and of being discontinued or deregistered 0.046.

The resulting probability vector $\underline{P_1}$ is as shown here $\begin{bmatrix} 0.782 \\ 0.126 \\ 0.046 \\ 0 \\ 0.046 \end{bmatrix}$

The matrix multiplication then yields:

$$\begin{bmatrix} 0.853 & 0.455 & 0 & 0 & 0 \\ 0.074 & 0.182 & 0.250 & 0 & 0 \\ 0.059 & 0.273 & 0.500 & 0 & 0 \\ 0 & 0 & 0.250 & 0 & 0 \\ 0.015 & 0.091 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.782 \\ 0.126 \\ 0.046 \\ 0 \\ 0.046 \end{bmatrix} = \begin{bmatrix} 0.724 \\ 0.092 \\ 0.103 \\ 0.011 \\ 0.069 \end{bmatrix}$$

This means that in the second year of study, the probability of passing all units was 0.724, of passing with one failed unit 0.092, of passing after taking supplementary examinations 0.103, of passing after repeating 0.011, and of being discontinued or deregistered 0.069. These results are based on the resulting probability vector $\underline{P_2}$ as shown above on them right hand side of the matrix equation.

A similar procedure was then applied for the progress (or lack of it) for students proceeding form the second to the third year of study, and for students proceeding form the third to the fourth year of study, as shown below.

4.2 Progress from Second to Third Year of Study

The corresponding table and matrix reflect the transition probabilities observed for students between their second and third years of study.

TABLE

	TRANS	MATRIX	YR 2 TO YR 3		PROB
	GIVENY2 >				
ACTUALY3]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.857	0.375	0.111	0	0
PASS 1F	0.063	0.250	0.111	0	0
PASS SUP	0.0476	0	0.556	1	0
PASS RP	0.0159	0.125	0.222	0	0

MATRIX

$$\begin{bmatrix} 0.857 & 0.375 & 0.111 & 0 & 0 \\ 0.063 & 0.250 & 0.111 & 0 & 0 \\ 0.047 & 0 & 0.556 & 1 & 0 \\ 0.016 & 0.125 & 0.222 & 0 & 0 \\ 0.016 & 0.250 & 0 & 0 & 1 \end{bmatrix}$$

Pre-multiplication by this transition matrix, on the probability vector $\underline{P}_2$ for a student being in one of the five states achieved after the second year of study, will yield the corresponding probability vector $\underline{P}_3$ for a student being in one of these five states after the third year of study. The matrix multiplication then yields:

$$\begin{bmatrix} 0.857 & 0.375 & 0.111 & 0 & 0 \\ 0.063 & 0.250 & 0.111 & 0 & 0 \\ 0.047 & 0 & 0.556 & 1 & 0 \\ 0.016 & 0.125 & 0.222 & 0 & 0 \\ 0.016 & 0.250 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.724 \\ 0.092 \\ 0.103 \\ 0.011 \\ 0.069 \end{bmatrix} = \begin{bmatrix} 0.667 \\ 0.080 \\ 0.103 \\ 0.046 \\ 0.103 \end{bmatrix}$$

This means that in the third year of study, the probability of passing all units was 0.667, of passing with one failed unit 0.080, of passing after taking supplementary examinations 0.103, of passing after repeating 0.046, and of being discontinued or deregistered 0.103. These results are based on the resulting probability vector $\underline{P}_3$ as shown above on the right hand side of the matrix equation. Sadly, by the end of third year, over 10% of the cohort were now discontinued or deregistered.

4.3 Progress from Third to Fourth Year of Study

The corresponding table and matrix reflect the transition probabilities observed for students between their third and fourth years of study.

TABLE

GIVENY3 > ACTUALY4]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.966	0.571	0.667	0	0
PASS 1F	0	0.286	0	0.250	0
PASS SUP	0.017	0	0.222	0	0

PASS RP	0	0	0.111	0.500	0
DIS/DR	0.017	0.143	0	0.250	1
TOTAL	1	1	1	1	1

MATRIX

$$\begin{bmatrix} 0.966 & 0.571 & 0.667 & 0 & 0 \\ 0 & 0.286 & 0 & 0.250 & 0 \\ 0.017 & 0 & 0.222 & 0 & 0 \\ 0 & 0. & 0.111 & 0.500 & 0 \\ 0.017 & 0.143 & 0 & 0.250 & 1 \end{bmatrix}$$

Pre-multiplication by this transition matrix, on the probability vector $\underline{P}_3$ for a student being in one of the five states achieved after the third year of study, will yield the corresponding probability vector $\underline{P}_4$ for a student being in one of these five states after the fourth year of study. The matrix multiplication then yields:

$$\begin{bmatrix} 0.966 & 0.571 & 0.667 & 0 & 0 \\ 0 & 0.286 & 0 & 0.250 & 0 \\ 0.017 & 0 & 0.222 & 0 & 0 \\ 0 & 0. & 0.111 & 0.500 & 0 \\ 0.017 & 0.143 & 0 & 0.250 & 1 \end{bmatrix} \begin{bmatrix} 0.667 \\ 0.080 \\ 0.103 \\ 0.046 \\ 0.103 \end{bmatrix} = \begin{bmatrix} 0.759 \\ 0.034 \\ 0.034 \\ 0.034 \\ 0.138 \end{bmatrix}$$

This means that in the fourth year of study, the probability of passing all units was 0.759, of passing with one failed unit 0.034, of passing after taking supplementary examinations 0.034, of passing after repeating 0.034, and of being discontinued or deregistered 0.138. These results are based on the resulting probability vector $\underline{P}_4$ as shown above on the right hand side of the matrix equation. Sadly, by the end of the final year, nearly 14% of the cohort were now discontinued or deregistered.

4.3 Students Admitted to B Sc Actuarial Science in JKUAT May 2006 Progress from First to Second Year of Study

The corresponding table and matrix reflect the transition probabilities observed for students between their first and second years of study.

TABLE

	TRANS GIVENY > 1	MATRIX YR 1 TO YR 2		PROB	
ACTUA LY2]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS	0.873016	0.5	0.285714	0	0

ALL					
PASS 1F	0.063492	0.277778	0.428571	0	0
PASS SUP	0.015873	0.166667	0	0	0
PASS RP	0.015873	0	0	0	0
DIS/DR	0.031746	0.055556	0.285714	0	1
TOTAL	1	1	1	0	1

MATRIX

$$\begin{bmatrix} 0.873 & 0.500 & 0.286 & 0 & 0 \\ 0.063 & 0.278 & 0.429 & 0 & 0 \\ 0.016 & 0.167 & 0 & 0 & 0 \\ 0.016 & 0. & 0 & 0 & 0 \\ 0.031 & 0.056 & 0,286 & 0 & 1 \end{bmatrix}$$

In the first year of study, the probability of passing all units was 0.643, of passing with one failed unit 0.184, of passing after taking supplementary examinations 0.071, of passing after repeating 0, and of being discontinued or deregistered 0.102.
The resulting probability vector $\underline{P}_1$ is as shown below.

$$\begin{bmatrix} 0.643 \\ 0.184 \\ 0.071 \\ 0 \\ 0.102 \end{bmatrix}$$

Pre-multiplication by this transition matrix, on the probability vector $\underline{P}_1$for a student being in one of the five states achieved after the first year of study, will yield the corresponding probability vector $\underline{P}_2$ for a student being in one of these five states after the second year of study. The matrix multiplication then yields:

$$\begin{bmatrix} 0.873 & 0.500 & 0.286 & 0 & 0 \\ 0.063 & 0.278 & 0.429 & 0 & 0 \\ 0.016 & 0.167 & 0 & 0 & 0 \\ 0.016 & 0. & 0 & 0 & 0 \\ 0.031 & 0.056 & 0,286 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.643 \\ 0.184 \\ 0.071 \\ 0 \\ 0.102 \end{bmatrix} = \begin{bmatrix} 0.673 \\ 0.122 \\ 0.041 \\ 0.010 \\ 0.153 \end{bmatrix}$$

This means that in the second year of study, the probability of passing all units was 0.673, of passing with one failed unit 0.122, of passing after taking supplementary examinations 0.041, of passing after repeating 0.010, and of being discontinued or deregistered 0.153. These results are based on the resulting probability vector $\underline{P}_2$ as shown above on the right hand side of the matrix equation. Sadly, by the end of second year, over 15% of the cohort were now discontinued or deregistered.

4.4 Progress from Second to Third Year of Study

The corresponding table and matrix reflect the transition probabilities observed for students between their second and third years of study.

TABLE

ACTUAL Y3]	TRANS GIVENY 2	MATRIX YR 2 TO YR 3 >			PROB
	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.833333	0.25	0.25	1	0
PASS 1F	0.106061	0	0.25	0	0
PASS SUP	0.060606	0.333333	0.25	0	0
PASS RP	0	0	0.25	0	0
DIS/DR	0	0.416667	0	0	1

MATRIX

$$\begin{bmatrix} 0.833 & 0.250 & 0.250 & 1 & 0 \\ 0.106 & 0 & 0.250 & 0 & 0 \\ 0.061 & 0.333 & 0.250 & 0 & 0 \\ 0 & 0 & 0.250 & 0 & 0 \\ 0 & 0.416 & 0,286 & 0 & 1 \end{bmatrix}$$

Pre-multiplication by this transition matrix, on the probability vector $\underline{P}_2$ for a student being in one of the five states achieved after the second year of study, will yield the corresponding probability vector $\underline{P}_3$ for a student being in one of these five states after the third year of study. The matrix multiplication then yields:

$$\begin{bmatrix} 0.833 & 0.250 & 0.250 & 1 & 0 \\ 0.106 & 0 & 0.250 & 0 & 0 \\ 0.061 & 0.333 & 0.250 & 0 & 0 \\ 0 & 0 & 0.250 & 0 & 0 \\ 0 & 0.416 & 0,286 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.673 \\ 0.122 \\ 0.041 \\ 0.010 \\ 0.153 \end{bmatrix} = \begin{bmatrix} 0.612 \\ 0.082 \\ 0.092 \\ 0.010 \\ 0.204 \end{bmatrix}$$

This means that in the third year of study, the probability of passing all units was 0.612, of passing with one failed unit 0.082, of passing after taking supplementary examinations 0.092, of passing after repeating 0.010, and of being discontinued or deregistered 0.204. These results are based on the resulting probability vector $\underline{P}_3$ as shown above on the right hand side of the matrix equation. Sadly, by the end of third year, over 20% of the cohort were now discontinued or deregistered.

4.5 Progress from Third to Fourth Year of Study

The corresponding table and matrix reflect the transition probabilities observed for students between their third and fourth years of study.

TABLE

	TRANS GIVENY 3	MATRIX YR 3 TO YR 4 >			PROB
ACTUA LY4]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.866667	0.625	0.333333	1	0
PASS 1F	0.066667	0	0.222222	0	0
PASS SUP	0.05	0.25	0.333333	0	0
PASS RP	0.016667	0	0.111111	0	0
DIS/DR	0	0.125	0	0	1

MATRIX

$$\begin{bmatrix} 0.867 & 0.625 & 0.333 & 1 & 0 \\ 0.067 & 0 & 0.222 & 0 & 0 \\ 0.050 & 0.250 & 0.333 & 0 & 0 \\ 0.017 & 0 & 0.111 & 0 & 0 \\ 0 & 0.125 & 0 & 0 & 1 \end{bmatrix}$$

Pre-multiplication by this transition matrix, on the probability vector $\underline{P}_3$ for a student being in one of the five states achieved after the third year of study, will yield the corresponding probability vector $\underline{P}_4$ for a student being in one of these five states after the fourth year of study.

The matrix multiplication then yields:

$$\begin{bmatrix} 0.867 & 0.625 & 0.333 & 1 & 0 \\ 0.067 & 0 & 0.222 & 0 & 0 \\ 0.050 & 0.250 & 0.333 & 0 & 0 \\ 0.017 & 0 & 0.111 & 0 & 0 \\ 0 & 0.125 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.612 \\ 0.082 \\ 0.092 \\ 0.010 \\ 0.204 \end{bmatrix} = \begin{bmatrix} 0.622 \\ 0.061 \\ 0.082 \\ 0.020 \\ 0.214 \end{bmatrix}$$

This means that in the fourth year of study, the probability of passing all units was 0.622, of passing with one failed unit 0.061, of passing after taking supplementary examinations 0.082, of passing after repeating 0.020, and of being discontinued or deregistered 0.214. These results are based on the resulting probability vector $\underline{P}_4$ as shown above on the right hand side of the matrix equation. Sadly, by the end of the final year, over 21% of the cohort were now discontinued or deregistered.

4.6 Proposed Transition Matrices for B Sc Actuarial Science Progress from First to Second Year of Study

TABLE

GIVENY1 > ACTUALY2]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.863	0.483	0.182	0	0
PASS 1F	0.069	0.241	0.364	0	0
PASS SUP	0.038	0.207	0.182	0	0
PASS RP	0.008	0	0.091	0	0
DIS/DR	0.023	0.069	0.182	0	1
TOTAL	1	1	1	0	1

MATRIX

$$\begin{bmatrix} 0.863 & 0.483 & 0.182 & 0 & 0 \\ 0.069 & 0.241 & 0.364 & 0 & 0 \\ 0.038 & 0.207 & 0.182 & 0 & 0 \\ 0.008 & 0 & 0.091 & 0 & 0 \\ 0.023 & 0.069 & 0.182 & 0 & 1 \end{bmatrix}$$

A sad implication of this matrix is seen in the last row, where there is a small probability of 0.023, that a student who has passed all units in first year, will be discontinued or deregistered after second year, a probability of 0.069, that a student who has passed with one failed unit in first year, will be discontinued or deregistered after second year, and a disturbing probability of 0.182, that a student who has passed after taking supplementary examinations in first year, will be discontinued or deregistered after second year. On the positive side, there is a probability of 0.863 that a student who has passed all units at the first attempt in first year, will also pass all units at the first attempt in the second year.

4.7 Progress from Second to Third Year of Study

TABLE

GIVENY2 >

ACTUALY3

]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.845	0.300	0.154	0.5	0
PASS 1F	0.085	0.100	0.154	0	0
PASS SUP	0.054	0.200	0.462	0.5	0
PASS RP	0.008	0.050	0.231	0	0
DIS/DR	0.008	0.350	0.000	0	1
TOTAL	1	1	1	1	1

MATRIX

$$\begin{bmatrix} 0.845 & 0.300 & 0.154 & 0.500 & 0 \\ 0.085 & 0.100 & 0.154 & 0 & 0 \\ 0.054 & 0.200 & 0.462 & 0.500 & 0 \\ 0.008 & 0.050 & 0.231 & 0 & 0 \\ 0.008 & 0.350 & 0 & 0 & 1 \end{bmatrix}$$

A sad implication of this matrix is seen in the last row, where there is a small probability of 0.008, that a student who has passed all units in second year, will be discontinued or deregistered after third year, a probability of 0.350, that a student who has passed with one failed unit in second year, will be discontinued or deregistered after third year. On the positive side, there is a probability of zero, that a student who had passed second year after taking supplementary examinations, would be discontinued or deregistered after third year, and also a probability of zero, that a student who had passed second year after repeating the year, would be discontinued or deregistered after third year. There is a high probability of 0.845 that a student who has passed all units at the first attempt in the second year, will also pass all units at the first attempt in the third year.

4.8 Progress from Third to Fourth Year of Study

TABLE

GIVENY3 >

ACTUALY4

]	PASS ALL	PASS 1F	PASS SUP	PASS RP	DIS/DR
PASS ALL	0.915	0.600	0.500	0.2	0
PASS 1F	0.034	0.133	0.111	0.2	0
PASS SUP	0.034	0.133	0.278	0	0

PASS					
RP	0.008	0.000	0.111	0.4	0
DIS/DR	0.008	0.133	0.000	0.2	1
TOTAL	1	1	1	1	1

MATRIX

$$\begin{bmatrix} 0.915 & 0.600 & 0.500 & 0.200 & 0 \\ 0.034 & 0.133 & 0.111 & 0.200 & 0 \\ 0.034 & 0.133 & 0.278 & 0 & 0 \\ 0.008 & 0. & 0.111 & 0.400 & 0 \\ 0.008 & 0.133 & 0 & 0.200 & 1 \end{bmatrix}$$

A sad implication of this matrix is seen in the last row, where there is a small probability of 0.008, that a student who has passed all units in third year, will be discontinued or deregistered after fourth year, a probability of 0.133, that a student who has passed with one failed unit in third year, will be discontinued or deregistered after fourth year, and a probability of 0.200, that a student who had passed third year after repeating the year, will be discontinued or deregistered after fourth year. On the positive side, there is a probability of zero, that a student, who had passed third year after taking supplementary examinations, would be discontinued or deregistered after fourth year, and a high probability of 0.915, that a student who has passed all units at the first attempt in the third year, will also pass all units at the first attempt in the fourth year.

5.0 Conclusions from the Study

At present, it is difficult to feel optimistic regarding the way forward to reducing the number of casualties, i.e. students who have to be discontinued or deregistered from their programme. In the case of students being deregistered, in later years, from the Actuarial science programme, this is sometimes as a result of the student being offered lucrative employment, even before completing the degree. In a few cases, a student may be deregistered due to medical problems, which do not permit the student to continue with the studies. In most cases, the cause is academic failure or lack of interest in the programme. Probably it would be advisable for a student, finishing Form 4 in November of year n, and receiving the KCSE results in March of year $n + 1$, to have a longer period out of school before embarking on a University programme in May of year $n + 1$. In some cases, it may be a case of a student being rushed into a programme, which with more time to think about it, the student did not really wish to pursue. If, of course, students are really set, in heart and mind, on the programme that they want, then there would be no harm in seeking admission for that programme in May of year $n + 1$. Possibly the proposal from the Ministry of Higher Education, that the intake for JAB (Government Sponsored) students in future be in September of year $n + 1$, for students finishing Form 4 in November of year n, would be the way forward, with the ADP intake at the JKUAT main campus also held back from May to November each year. A September intake would give students, who are uncertain about their future, adequate time to think about their options.

Perhaps some recommendations by Uppal and Humphreys in the May 2011 CULMS Community for Undergraduate Learning in the Mathematical Sciences), in their section on ‘The Way Forward’ of the paper ‘Teaching and Learning of Undergraduate Mathematics and Statistics

(Kenyan Scene)' would be worth considering, especially with regard to the students' attitude to units taught by service departments, for the sake of enhancing the programme run by the owning department.

REFERENCES

Burke, G. (1972), *A study in the economics of education with particular reference to the supply of secondary teachers for government schools in Victoria*. PhD thesis, Monash University.

Nicholls, M., G. (2009), *The use of Markov models as an aid to the evaluation, planning and benchmarking of Doctoral Programs* Journal of the Operational Research Society (2009) 60: 1183-1190

Uppal, S., M., and Humphreys, H., M. (2011), *Teaching and Learning of Undergraduate Mathematics and Statistics* (Kenyan Scene) CULMS Newsletter No. 3.

Gani, J. (1963), Formulae for projecting Enrolments and Degrees awarded in universities. *Journal of the Royal Statistical Society*, A126, pp. 400-409.

Odhiambo, J., W., and J., O., Owino. (1985), A Stochastic Model for Estimating Academic Survival in an Education System, *Kenya Journal of Science and Technology*, A6, pp. 159-76.

Odhiambo, J., W., and A., K., Khogali. (1986), A Transition Model for Estimating Academic Survival through Cohort Analysis. International Journal of Science and Technology, Vol 16, pp. 339-346

Owino, J., O. (1982), Markovian Model for Education Planning, M.Sc Research Project, submitted to the Department of Mathematics. University of Nairobi, Kenya.

Owino, J., O., and Odhiambo, J., W. (1994), *A Statistical Model for Planning an Education System Discovery and Innovation*, Vol 6, No. 2, pp. 140-144.

Owino, J., O., and Phillips, C., M. (1988), A Comparison of Retention Properties of the Kenyan Primary Education System before and after 1972. *Kenya Journal of Science*, A9, pp. 5-10.

Uche P. L. (1980) A Transition Model for Academic Survival in a single Channel System, *International Journal of Education, Science and Technology* Vol. 17, pp. 177-187

Appendix

Table 1

GIVEN YEAR1	PASS ALL UNITS	PASS ONE FAIL	PASS ON SUPP	PASS ON REPEAT	DISCONT/ DEREG	TOTAL
ACTUAL YEAR 2						
PASS ALL UNITS	58	5	0	0	0	63
PASS ONE FAIL	5	2	1	0	0	8
PASS ON SUPP	4	3	2	0	0	9
PASS ON REPEAT	0	0	1	0	0	1
DISCONT/ DEREG	1	1	0	0	4	6
TOTAL	68	11	4	0	4	87